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THE OPEN UNIVERSITY OF SRI LANKA
BACHELOR OF SOFTWARE ENGINEERING
DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING

ECZ3161 - MATHEMATICS FOR COMPUTING
FINAL EXAMINATION - 2009/ 10 CLOSE BOOK

Date: May 12, 2010 **Time: 0930 – 1230 hrs**

READ THE FOLLOWING INSTRUCTIONS BEFORE ANSWERING THE PAPER

Instructions

1. Answer any **five** questions.
2. Show all steps clearly.
3. Do **not** allow the **programmable calculators.**

Q1

(a) Use Demorgan's theorem to simplify the following expressions:

i) $\overline{(\bar{a}+b+c)bc}$ ii) $\overline{(\bar{c}+\bar{a}+b)(\bar{a}+b)}$

iii) $\overline{(x+\bar{y}+z)}+\overline{(\bar{x}+y+z)}+\overline{(x+y+\bar{z})}$

(b) Use Truth tables to show the followings:

i) $\overline{(\bar{a}+\bar{b})(\bar{a}+b)}=a$ ii) $(x+\bar{y})(\bar{x}+y)=xy+\overline{x+y}$

(c) Consider the following truth table.

A	B	C	D	Result
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	0
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

i) Setup the Karnaugh map for the above truth table.

ii) Then find the solution and simplify using the K map.

Q2

(a) General $n \times n$ matrix is given by A.

$$A = \begin{pmatrix} 2 & 1 & 3 \\ -2 & 0 & 1 \\ 3 & 1 & 2 \end{pmatrix}$$

Find

i) $A + A^t$

ii) $A - A^t$

(b) Let the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ a & -1 & 0 \\ b & c & 1 \end{pmatrix}$$

i) Find A^2.

ii) For what relation between a, b and c is $A^2 = I$ (the unit matrix)?

Q3

(a) If A and B are both $n \times n$ matrices with A non-singular, show that

$$(A^{-1}BA)^2 = A^{-1}B^2A$$

(b) Consider the following matrix A.

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 4 & 9 \end{pmatrix}$$

i) Find the inverse of the matrix using simplifying method.

ii) Hence, solve the following linear equations when $\lambda = 3$:

$$x + y + z = 5$$
$$x + 2y + \lambda z = 13$$
$$x + 4y + \lambda^2 z = 35$$

Q4

(a) Given that $Tan\alpha = 3/4$, α in quadrant I, and $Cos\beta = 1/2$, β in quadrant IV, Find

i) $Cos(\alpha + \beta)$ ii) $Sin2\alpha$ iii) $Tan(\alpha - \beta)$

Give exact answers and show all your work.

(b) Sketch the graph $y = 3\cos\left(x - \frac{\pi}{4}\right)$ in the period of $-2\pi \le x \le 2\pi$.

(c) Answer the following problems:

i) A ship of height h = 15 m is sighted from a lighthouse. From the top of the lighthouse, the angle to the top of the mast and the base of the ship equal 30° and 45° respectively. How far is the ship from the lighthouse? (Assume that the base of the lighthouse and the base of the ship at the same level.)

ii) You are stationed at a radar base and you observe an unidentified plane at an altitude h = 5000 m flying towards your radar base at an angle of elevation = 30°. After exactly one minute, your radar sweep reveals that the plane is now at an angle of elevation = 60° maintaining the same altitude. What is the speed (in m/s) of the plane?

Q5

(a) Answer the followings, show all your work.

i) Write an equation for $Sin(A+B)$

ii) Prove that $Sin3A = 3SinA - 4Sin^3A$ using above i)

iii) Hence, find the value of $Sin135°$

(b) Prove the followings.

i) $(\cos A + \cos B)^2 + (\sin A - \sin B)^2 = 2(1+\cos(A+B))$

ii) $\dfrac{\sin 2x - \cos x}{\sin x - \cos 2x} = \dfrac{\cos x}{\sin x + 1}$

(c) Find the general solution of the following equation for the range $0° < \theta < 360°$.

$$\sin^2\theta + 2\cos\theta = -2$$

Q6

(a) Find the following limits:

i) $\lim\limits_{x\to 0} \dfrac{Sin6x}{5x}$ ii) $\lim\limits_{x\to 4} \dfrac{2x^3-128}{\sqrt{x}-2}$ iii) $\lim\limits_{x\to -2} \dfrac{\frac{1}{2}+\frac{1}{x}}{x^3+8}$

(b) A ball is thrown into air with a velocity of $40ms^{-1}$. Its height in meters after t seconds is given by $y(t) = 40t - 16t^2$.

i) By using the formula $\lim\limits_{h\to 0} \dfrac{y(t+h)-y(t)}{h}$ find the velocity of the ball.

ii) Hence find the velocity after 3 seconds.

iii) The time taken by the ball to return to the ground.

(c) The difference table for $f(x) = e^x$ with $h = 0.2$ is shown below.

x_j	f_j	Δf_j	$\Delta^2 f_j$	$\Delta^3 f_j$	$\Delta^4 f_j$
0.0	1.0000				
		0.2214			
0.2	1.2214		0.0490		
		0.2704		0.0109	
0.4	1.4918		0.0599		0.0023
		0.3303		0.0132	
0.6	1.8221		0.0731		0.0031
		0.4034		0.0163	
0.8	2.2255		0.0894		0.0033
		0.4928		0.0196	
1.0	2.7183		0.1090		0.0047
		0.6018		0.0243	
1.2	3.3201		0.1333		
		0.7351			
1.4	4.0552				

Select $x_0 = 0.6$ and find the value of $f(0.63) = e^{0.63}$ by using Newton's forward formula.

Q7

(a) Find the first derivatives of the following from first principles. Show all steps.

i) $1/\sqrt{x}$ ii) $Tan\,x$

(b) Find $\frac{dy}{dx}$ as a function of x for

i) $y=(2-x^2)^{1/3}$ ii) $y=\frac{x^3(x+1)}{(3x-2)^4}$

(c) If $y = xSin\,x$ show that

$$\left(\frac{dy}{dx}\right)^2-\left(\frac{dy}{dx}\right)Sin\,x+y^2=x\left(x+\frac{Sin\,2x}{2}\right)$$

Q8

(a) Evaluate the following.

i) $\int(Sin\,2x+Cos\,3x)\,dx$ ii) $\int 4Sin(3-4x)\,dx$

(b) Find the exact value of the following.

i) $\int_0^1 \frac{1}{\sqrt{4-3x^2}}\,dx$ ii) $\int_0^1 \frac{1}{3+x^2}\,dx$

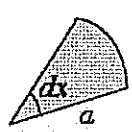
(b) Evaluate the following.

i) $\int \log|x|\,dx$ ii) $\int x e^{-3x}\,dx$

(d) Radii of the inner and outer circles of a tire are a and b respectively.

i) Write an equation to find the cross sectional area of the tire <u>by integration</u> method. (Use the following Hint)

<u>Hint:</u>



If a is the radius and dx is the angle of the strip then,

The area of the strip is $\frac{1}{2}a^2\,dx$

ii) Hence, find the cross sectional area of the tire.

- End.