THE OPEN UNIVERSITY OF SRI LANKA
BACHELOR OF SOFTWARE ENGINEERING
DEPARTMENT OF ELECTRICAL & COMPUTER ENGINEERING

ECZ3161 – MATHEMATICS FOR COMPUTING
FINAL EXAMINATION – 2012/13 **CLOSED BOOK**

Date: July 24, 2013 **Time: 09.30-12.30 hrs**

Instructions

1. Answer any **five** out of eight questions.
2. Show all steps clearly.
3. **Programmable** calculators are **not** allowed.

Q1

(a) Use Boolean algebra to verify the following. **(4x2 marks)**

i) $\bar{a}\bar{b}\bar{c} + a\bar{b}\bar{c} + \bar{a}b\bar{c} + abc = \overline{\bar{a}\bar{b}\bar{c} + a\bar{b}\bar{c} + \bar{a}b\bar{c} + abc}$

ii) $\overline{\bar{a}\bar{b}cd + abcd + a\bar{b}cd + abc\bar{d}} = \bar{c} + \bar{a}\bar{b} + \bar{b}\bar{d} + \bar{a}\bar{d}$

(b) Use Truth tables to show the followings. **(3x2 marks)**

i) $xy + x\bar{z} = xyz + xy\bar{z} + x\bar{y}\bar{z}$

ii) $\overline{\bar{x} + y} + \bar{x}\bar{y} = x\bar{y} + \bar{x}\bar{y}$

(c) Use Karnaugh map and find minimal sum for the followings. **(3x2 marks)**

i) $\bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c + a\bar{b}\bar{c} + abc$

ii) $xyz + x\bar{y}z + \bar{x}yz + \bar{x}y\bar{z} + \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z$

Q2

(a)

If $A = \begin{pmatrix} 4 & 2 \\ -1 & 1 \end{pmatrix}$, show that

$A^2 + 6I = 5A$; where I is the identity matrix of order 2. **(4 marks)**

(b)

Let $A = \begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}$, show that $A^2 = A$

Hence deduce that $(I - A)^2 = (I - A)$, where I is the identity matrix of order 3.

(6 marks)

(c) The matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ are given by

$$\boldsymbol{A}=\begin{pmatrix}1 & 2\\ 3 & 4\end{pmatrix} \text{ and } \boldsymbol{B}=\begin{pmatrix}\lambda & 0\\ 0 & \mu\end{pmatrix}$$

Find ABA^{-1} and write down $AA^{-1}B$.
Explain why you would normally expect these results to be different.
Show that when λ and μ such that ABA^{-1} is diagonal then ABA^{-1} reduces to $\boldsymbol{B}$. **(10 marks)**

Q3 Consider 3×3 matrix A,

$$A=\begin{pmatrix}0 & 1 & 2\\ 1 & 2 & 3\\ 3 & 1 & 1\end{pmatrix}$$

(a) Find AA^T **(6 marks)**

(b) Find the inverse of the matrix A using Gausian elimination method. **(14 marks)**

Q4

(a) Given that $\tan\theta=\frac{3}{4}$, θ in quadrant I, and $\tan\alpha=\frac{40}{9}$, α in quadrant I. Find

i) $\sin(\theta+\alpha)$ ii) $\cos(\alpha-\theta)$ iii) $\tan(\theta-\alpha)$ **(2x3 marks)**

Give exact answers and show all your work.

(b) Sketch the graph of $y=\cos^2 x$ in the period $-2\pi\le x\le 2\pi$. **(6 marks)**

(c) Answer the following problems

i) From the top of a lighthouse, 120m above the sea, the angle of depression (downward angle from horizontal direction) of a boat is 15°. How far is the boat from the lighthouse? **(3 marks)**

ii) Find the height of a tree if the angle of elevation of its top changes from 20° to 40° as the observer advances 75m toward its base. **(5 marks)**

Q5

(a) Let $p=\operatorname{cosec}\theta+\cot\theta$, where θ is not an even multiple of π and $p\neq 0$.

Show that, $\operatorname{cosec}\theta-\cot\theta=\frac{1}{p}$

Deduce that, $\sin\theta=\frac{2p}{1+p^2}$ and $\cos\theta=-\frac{1-p^2}{1+p^2}$ **(4 marks)**

(b) Prove the following. **(4x3 marks)**

i) $\dfrac{\sin(A+B)+\sin(A-B)}{\cos(A+B)-\cos(A-B)}=-\cot B$

ii) $\sin^2 2x=(2\tan x-\sin 2x)(2\cot x-\sin 2x)$

iii) $\dfrac{\sin 75^\circ + \sin 15^\circ}{\cos 75^\circ + \cos 15^\circ} = 1$

(c) Find the general solution of the following equation **(4 marks)**
$1 + 4\cos^2\theta - 5\cos\theta - \cos 2\theta = 0$ in the range $0^0 \le \theta \le 360^0$.

Q6

(a) Find the following limits

i) $\lim\limits_{x\to 0} \dfrac{\sin x + x}{\tan x}$

ii) $\lim\limits_{x\to 3} \dfrac{\sqrt{x+1} - 2}{x^2 - 9}$

iii) $\lim\limits_{x\to 0} \dfrac{(x-1)^2 - 1}{x(x-1)}$ **(4x3 marks)**

(b) i) Starting with $x_0 = 3$ find a root of $x^3 - 3x - 5 = 0$, correct to three decimal places. Use Newton-Raphson method.

ii) By using Newton-Raphson method, find the root of $x^4 - x - 10 = 0$, which is near to $x = 2$ correct to three places of decimal. **(4x2 marks)**

Q7

(a) Find first derivatives of the following from first principles. Show all steps. **(3x2 marks)**

i) $x^2 + 2$ ii) $\sin x + 1$

(b) Find $\dfrac{dy}{dx}$ as a function of x for

i) $y = \dfrac{1}{3}\left(\sqrt{1+x^4} - x^2\right)^3$ ii) $y = x^2 \sin x$ **(4 x2marks)**

(c) If $y = -3x - \dfrac{1}{2}\sin 2x + 4\cos x$ show that **(6 marks)**

$\dfrac{dy}{dx} = -6 + 2(\sin x - 1)^2$

Q8

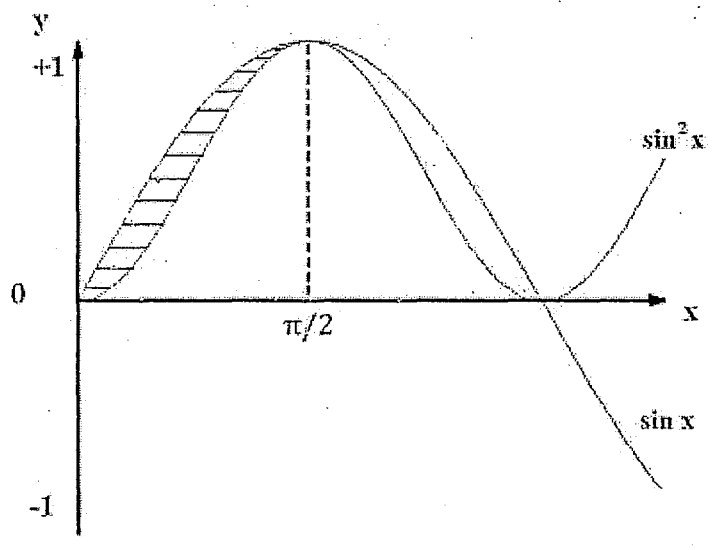
(a) Evaluate the following. **(3x2 marks)**

i) $\int (\cos x - \sin x)dx$ ii) $\int \sin(1-x)dx$

(b) Find the exact value of the following. **(3x2 marks)**

i) $\int_0^1 \dfrac{1}{x^2+1}dx$ ii) $\int_0^3 \dfrac{1}{\sqrt{9-x^2}}dx$

(c) Consider the following figure with two curves.



i) Write an equation to find the shaded area of the figure by integration method. **(4 marks)**

ii) Hence, find the shaded area of the figure. **(4 marks)**