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The Open University of Sri Lanka
B.Sc/B.Ed. Degree Programme
Final Examination - 2009/2010
Applied Mathematics - Level 03
APU1140 – Vector Algebra
Duration: - Two hours

Date: 20.01.2010 Time: 9.30 a.m. - 11.30 a.m.

Answer **Question No. 6** and **three** other questions.

1. (a) Let $OAB$ be a triangle and let $X$ and $Y$ be the mid- points of $OA$ and $OB$ respectively. $Z$ is the point of intersection of $XB$ and $YA$.
If $\overline{OA} = \underline{a}$ and $\overline{OB} = \underline{b}$, show that the position vector of $Z$ with respect to the origin $O$ is $\frac{1}{3}(\underline{a}+\underline{b})$.

(b) With respect to the origin $O$, the position vectors of the points $A, B, C$ and $D$ are $\underline{i}+\underline{j}$, $2\underline{i}+3\underline{j}$, $p\underline{i}+2\underline{j}$ and $\underline{i}+q\underline{j}$ respectively, where $p$ and $q$ are real constants. It is given that $ABCD$ parallelogram. Without using scalar products or vector products,
(i) Find $p, q$ and the angle $A\hat{D}C$,
(ii) Show that $ABCD$ is not a rhombus.

2. Define (i) the scalar product $\underline{a}\cdot\underline{b}$ and (ii) the vector product $\underline{a}\times\underline{b}$ of two given vectors $\underline{a}$ and $\underline{b}$.
Given that $A, B$ and $C$ are three non collinear points and $O$ is the origin, interpret the quantity $\left|\overline{OA}\times\overline{OB}\cdot\overline{OC}\right|$.
The position vectors $\underline{a}$, $\underline{b}$ and $\underline{c}$ of the three points $A, B$ and $C$ with respect to the origin $O$ are $\underline{i}+2\underline{j}+3\underline{k}$, $2\underline{i}+\underline{j}+\underline{k}$ and $\underline{i}-\underline{j}+2\underline{k}$ respectively.
Find
(i) the cosine of the angle $A\hat{B}C$.
(ii) the area of the triangle $ABC$.
(iii) the volume of the tetrahedron $OABC$.



3. (a) The position vectors of the points $P, Q, R$ are given by $\underline{i}+2\underline{j}+\underline{k}$, $3\underline{i}-\underline{j}+2\underline{k}$ and $4\underline{i}-\frac{5}{2}\underline{j}+\frac{5}{2}\underline{k}$ respectively.
(i) Show that $P$, $Q$ and $R$ are collinear, and find the ratio $PQ:QR$.
(ii) Find the vector equation of the line $PQ$.
(iii) Show that the line $l$ given by $\underline{r}=\frac{7}{2}\underline{i}-2\underline{j}+3\underline{k}+\lambda\left(-\underline{i}+\underline{j}+\underline{k}\right)$, passes through the point $R$, and find the acute angle between $l$ and the line $PQ$.

(b) Prove that, if four points $A, B, C, D$ with position vectors **a, b, c, d** are coplanar, then
$$[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]+[\mathbf{a}\ \mathbf{c}\ \mathbf{d}]=[\mathbf{a}\ \mathbf{b}\ \mathbf{d}]+[\mathbf{b}\ \mathbf{c}\ \mathbf{d}].$$
[Here $[\mathbf{x}\ \mathbf{y}\ \mathbf{z}]$ denotes the scalar triple product $\mathbf{x}\times\mathbf{y}\cdot\mathbf{z}$ of the three vectors $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$.]

4. Show that the distance of the point $E$ with position vector $\underline{e}$ from the plane with equation $\underline{r}\cdot\underline{n}=d$ is $\dfrac{|d-\underline{e}\cdot\underline{n}|}{|\underline{n}|}$.

Referred to a fixed origin $O$, the position vectors of the points $A$, $B$, $C$ and $D$ are $-\underline{j}+\underline{k}$, $2\underline{i}-\underline{j}+3\underline{k}$, $-\underline{i}-2\underline{j}+2\underline{k}$ and $7\underline{i}-4\underline{j}+2\underline{k}$ respectively.

(i) Find a vector which is perpendicular to the plane $ABC$.

(ii) Using the result in the first part of this question, show that the length of the perpendicular from $D$ to the plane $ABC$ is of length $2\sqrt{6}$.
[No credit will be given for other methods]

(iii) Show that the planes $ABC$ and $BCD$ are perpendicular.

(iv) Find the cosine of the acute angle between the line $BD$ and the plane $ABC$.

5. (a) The position vectors of three points $A$, $B$, $C$ with respect to an origin $O$ are $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ respectively. If $OA \perp BC$ and $OB \perp AC$, obtain two equations involving the vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.
Use them to show that $\mathbf{a}\cdot\mathbf{c}=\mathbf{b}\cdot\mathbf{c}$.
Deduce that
(i) $OC \perp AB$
(ii) $OA^2+BC^2=OB^2+CA^2$.

(b) Calculate the work done by the force $\underline{F}(x,y)=(x+y)^2\,\underline{i}+(x-y)^2\,\underline{j}$ on a particle moving from (1, 0) to (0, 1) along the circle $\underline{r}=\cos t\,\underline{i}+\sin t\,\underline{j}$.

6. Cartesian coordinates $(x,y,z)$ of a point $P$ on a space curve are given, in terms of a parameter $\theta$, by $x=c\cos\theta$, $y=c\sin\theta$, $z=(c\tan\alpha)\theta$, where $c$ and $\alpha$ are positive constants with $0<\alpha<\dfrac{\pi}{2}$.

The arc distance of $P$, measured from the point $(c,0,0)$ is denoted by $s$. Find $\dfrac{ds}{d\theta}$.

(i) Show that the unit tangent vector is $\hat{\mathbf{t}}=-\mathbf{i}\cos\alpha\sin\theta+\mathbf{j}\cos\alpha\cos\theta+\mathbf{k}\sin\alpha$.

(ii) Using the formula $\dfrac{d\hat{\mathbf{t}}}{ds}=\kappa\hat{\mathbf{n}}$, show that the curvature $\kappa$ at a point $P$ is $\dfrac{1}{c}\cos^2\alpha$, and find the principal normal $\hat{\mathbf{n}}$.

(iii) Show that the binormal $\hat{\mathbf{b}}=\mathbf{i}\sin\alpha\sin\theta-\mathbf{j}\sin\alpha\cos\theta+\mathbf{k}\cos\alpha$.

(iv) Using the formula $\dfrac{d\hat{\mathbf{b}}}{ds}=-\tau\hat{\mathbf{n}}$, find the torsion $\tau$ at $P$.

Verify the formula $\dfrac{d\hat{\mathbf{n}}}{ds}=\tau\hat{\mathbf{b}}-\kappa\hat{\mathbf{t}}$, for the above space curve.

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ශ්‍රී ලංකා විවෘත විශ්වවිද්‍යාලය
විද්‍යාවේදී/ අධ්‍යාපනවේදී උපාධි පාඨමාලාව
අවසාන පරීක්ෂණය - **2009/2010**
ව්‍යවහාරික ගණිතය - තුන්වන මට්ටම
**APU 1140** – දෛශික විජය

කාලය පැය දෙකයි.

දිනය: 2010. 01. 20 | වේලාව: පෙ.ව. **9.30** - පෙ.ව. **11.30** දක්වා.

* **6** වන ප්‍රශ්නයට සහ තවත් ප්‍රශ්න තුනකට පිළිතුරු සපයන්න.

**1.** (a) $OAB$ ත්‍රිකෝණයක් ලෙස ද $X$ සහ $Y$ යනු පිළිවෙලින් $OA$ සහ $OB$ වල මධ්‍ය ලක්ෂ්‍ය ලෙස ද ගනිමු. $Z$ යනු $XB$ සහ $YA$ වල ඡේදන ලක්ෂ්‍යය වේ. $\overline{OA} = \underline{a}$ සහ $\overline{OB} = \underline{b}$ නම්, $O$ මූල ලක්ෂ්‍යයට සාපේක්ෂව $Z$ හි පිහිටුම් දෛශිකය $\frac{1}{3}(\underline{a}+\underline{b})$ බව පෙන්වන්න.

(b) $O$ මූලයට අනුබද්ධව, $A, B, C$ සහ $D$ ලක්ෂ්‍යවල පිහිටුම් දෛශික පිළිවෙලින් $\underline{i}+\underline{j}$, $2\underline{i}+3\underline{j}$, $p\underline{i}+2\underline{j}$ සහ $\underline{i}+q\underline{j}$ වේ. මෙහි $p$ සහ $q$ යනු තාත්ත්වික නියත වේ. $ABCD$ යනු සමාන්තරාස්‍රයකි. අදිශ ගුණිතය හෝ දෛශික ගුණිතය භාවිතයෙන් තොරව,

(i) $p, q$ සහ $A\hat{D}C$ කෝණය සොයන්න.
(ii) $ABCD$ යනු රොම්බසයක් නොවන බව පෙන්වන්න.

**2.** $\underline{a}$ සහ $\underline{b}$ දෛශික දෙකෙහි **(i)** $\underline{a}\cdot\underline{b}$ අදිශ ගුණිතය සහ **(ii)** $\underline{a}\times\underline{b}$ දෛශික ගුණිතය අර්ථ දක්වන්න.

$A, B, C$ යනු ඒක රේඛීය නොවන ලක්ෂ්‍ය තුනක් යැයිද $O$ යනු මූල ලක්ෂ්‍යය යැයිද දී ඇත්නම් $\left|\overline{OA}\times\overline{OB}\cdot\overline{OC}\right|$ රාශිය විවරණය කරන්න.

$O$ මූල ලක්ෂ්‍යයට අනුබද්ධයෙන් $A, B$ සහ $C$ ලක්ෂ්‍ය තුනෙහි $\underline{a}$, $\underline{b}$ සහ $\underline{c}$ පිහිටුම් දෛශික පිළිවෙලින් $\underline{i}+2\underline{j}+3\underline{k}$, $2\underline{i}+\underline{j}+\underline{k}$ සහ $\underline{i}-\underline{j}+2\underline{k}$ වේ.

(i) $A\hat{B}C$ කෝණයේ කෝසයිනය,
(ii) $ABC$ ත්‍රිකෝණයේ වර්ගඵලය,
(iii) $OABC$ චතුස්තලයේ පරිමාව

සොයන්න.

**3.** (a) $P, Q, R$ ලක්ෂ්‍යවල පිහිටුම් දෛශික පිළිවෙලින් $\underline{i}+2\underline{j}+\underline{k}$, $3\underline{i}-\underline{j}+2\underline{k}$ සහ $4\underline{i}-\frac{5}{2}\underline{j}+\frac{5}{2}\underline{k}$ වේ.

(i) $P, Q, R$ ඒක රේඛීය බව පෙන්වා, $PR:QR$ අනුපාතය සොයන්න.
(ii) $PQ$ රේඛාවේ දෛශික සමීකරණය සොයන්න.
(iii) $\underline{r} = \frac{7}{2}\underline{i}-2\underline{j}+3\underline{k}+\lambda(-\underline{i}+\underline{j}+\underline{k})$ මගින් දෙනු ලබන $l$ රේඛාව, $R$ ලක්ෂ්‍යය හරහා යන බව පෙන්වා, $l$ සහ $PQ$ රේඛාව අතර සුළු කෝණය සොයන්න.

(b) $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$ පිහිටුම් දෛශික සහිත $A, B, C, D$ ලක්ෂ්‍ය හතර ඒකතල වේ නම්, එවිට

$$[\mathbf{a}\ \mathbf{b}\ \mathbf{c}]+[\mathbf{a}\ \mathbf{c}\ \mathbf{d}]=[\mathbf{a}\ \mathbf{b}\ \mathbf{d}]+[\mathbf{b}\ \mathbf{c}\ \mathbf{d}]$$

බව සාධනය කරන්න.

[ මෙහි $[\mathbf{x}\ \mathbf{y}\ \mathbf{z}]$ යන්නෙන් $\mathbf{x}, \mathbf{y}, \mathbf{z}$ දෛශික තුනෙහි $\mathbf{x}\times\mathbf{y}\cdot\mathbf{z}$ අදිශ ත්‍රිත්ව ගුණිතය හැඳින්වේ.]

4. $\underline{r}\cdot\underline{n}=d$ සමීකරණය සහිත තලයේ සිට පිහිටුම් දෛශික $\underline{e}$ වූ $E$ ලක්ෂ්‍යයට ඇති දුර $\dfrac{|d-\underline{e}\cdot\underline{n}|}{|\underline{n}|}$ බ
පෙන්වන්න.

$O$ අචල මූලයක් අනුබද්ධව, $A, B, C$ සහ $D$ ලක්ෂ්‍යවල පිහිටුම් දෛශික පිළිවෙලින් $-\underline{j}+\underline{k},\ 2\underline{i}-\underline{j}+3\underline{k},\ -\underline{i}-2\underline{j}+2\underline{k}$ සහ $7\underline{i}-4\underline{j}+2\underline{k}$ වේ.

(i) $ABC$ තලයට ලම්භක දෛශිකයක් සොයන්න.

(ii) මෙම ප්‍රශ්නයෙහි පළමු කොටසෙහි වූ ප්‍රතිඵලය භාවිතයෙන් $D$ ලක්ෂ්‍යයේ සිට $ABC$ තලයට ඇ
ලම්භ දුර ඒකක $2\sqrt{6}$ බව පෙන්වන්න.
[වෙනත් ක්‍රම සඳහා ලකුණු ප්‍රදානය නොකෙරේ.]

(iii) $ABC$ සහ $BCD$ තල එකිනෙක ලම්භක බව පෙන්වන්න.

(iv) $BD$ රේඛාව සහ $ABC$ තලය අතර සුළු කෝණයේ කෝසයිනය සොයන්න.

5. (a) $O$ මූල ලක්ෂ්‍යයට අනුබද්ධයෙන් $A, B, C$ ලක්ෂ්‍ය තුනෙක පිහිටුම් දෛශික පිළිවෙලින් $\mathbf{a}, \mathbf{b}, \mathbf{c}$ වේ. $OA\perp BC$ සහ $OB\perp AC$ නම්, $\mathbf{a}, \mathbf{b}, \mathbf{c}$ දෛශික අඩංගු සමීකරණ දෙකක් ලබා ගන්න. ඒවා භාවිතයෙන් $\mathbf{a}\cdot\mathbf{c}=\mathbf{b}\cdot\mathbf{c}$ බව පෙන්වන්න. එනයින්,

(i) $OC\perp AB$,

(ii) $OA^2+BC^2=OB^2+CA^2$

බව අපෝහනය කරන්න.

(b) $\underline{F}(x,y)=(x+y)^2\,\underline{i}+(x-y)^2\,\underline{j}$ බලය මගින්, අංශුවක් $\underline{r}=\cos t\,\underline{i}+\sin t\,\underline{j}$ වෘත්තය දිගේ $(1,0)$ සිට $(0,1)$ දක්වා චලනය කිරීමේදී කරන කාර්යය ගණනය කරන්න.

6. අවකාශ වක්‍රයක් මත $P$ ලක්ෂ්‍යයක $(x,y,z)$ කාටීසියානු ඛණ්ඩාංක, $\theta$ පරාමිතියක් ඇසුරෙන් $x=c\cos\theta,\ y=c\sin\theta,\ z=(c\tan\alpha)\theta$ මගින් දී ඇත. මෙහි $c$ සහ $\alpha$ යනු ධන තාත්ත්වික නියත වන අතර $0<\alpha<\pi/2$ වේ.

$(c,0,0)$ ලක්ෂ්‍යයේ සිට $P$ ට ඇති චාප දුර $s$ මගින් අංකනය කර ඇත. $\dfrac{ds}{d\theta}$ සොයන්න.

(i) ඒකක ස්පර්ශකය $\hat{\mathbf{t}}=-\mathbf{i}\cos\alpha\sin\theta+\mathbf{j}\cos\alpha\cos\theta+\mathbf{k}\sin\alpha$ බව පෙන්වන්න.

(ii) $\dfrac{d\hat{\mathbf{t}}}{ds}=\kappa\hat{\mathbf{n}}$ සූත්‍රය භාවිතයෙන්, $P$ ලක්ෂ්‍යයෙහි වක්‍රතාව $\kappa$, $\dfrac{1}{c}\cos^2\alpha$ බව පෙන්වා, ප්‍රධාන අභිලම්බය, $\hat{\mathbf{n}}$ සොයන්න.

(iii) අපර අභිලම්බය $\hat{\mathbf{b}}=\mathbf{i}\sin\alpha\sin\theta-\mathbf{j}\sin\alpha\cos\theta+\mathbf{k}\cos\alpha$ බව පෙන්වන්න.

(iv) $\dfrac{d\hat{\mathbf{b}}}{ds}=-\tau\hat{\mathbf{n}}$ සූත්‍රය භාවිතයෙන්, $P$ හිදී ව්‍යාවර්තනය, $\tau$ සොයන්න.

ඉහත අවකාශ වක්‍රය සඳහා, $\dfrac{d\hat{\mathbf{n}}}{ds}=\tau\hat{\mathbf{b}}-\kappa\hat{\mathbf{t}}$ සූත්‍රය සත්‍යාපනය කරන්න.

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இலங்கை திறந்த பல்கலைக்கழகம்
விஞ்ஞானமாணிப் பட்டப் பாடநெறி
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**06** ஆம் வினாவிற்கும் ஏனைய மூன்று வினாக்களுக்கும் விடையளிக்குக.

**1.** (a) $OAB$ ஐ ஒரு முக்கோணியாகக் கொள்வதோடு, $X$ மற்றும் $Y$ என்பன முறையே $OA$ மற்றும் $OB$ என்பன நடுப்புள்ளிகளாகுமெனவும் கொள்க. $Z$ ஆனது $XB$ மற்றும் $YA$ என்பனவற்றின் இடைவெட்டுப் புள்ளியாகும். $\overline{OA}=\underline{a}$ மற்றும் $\overline{OB}=\underline{b}$ ஆகுமாயின், உற்பத்தி குறித்து $Z$ இன் தானக்காவியானது $\frac{1}{3}(\underline{a}+\underline{b})$ எனக் காட்டுக.

(b) உற்பத்தி **O** குறித்து, $A,B,C$ மற்றும் $D$ ஆகியவற்றின் தானக்காவிகள் முறைய $\underline{i}+\underline{j}$, $2\underline{i}+3\underline{j}$, $p\underline{i}+2\underline{j}$ மற்றும் $\underline{i}+q\underline{j}$ என்பனவாகும், இங்கு $p$ மற்றும் $q$ என்பன மெய் மாறிலிகளாகும். மேலும் $ABCD$ ஆனது இணைகரம் எனத் தரப்பட்டுள்ளது. எண்ணிப் பெருக்கங்களையோ அல்லது காவிப் பெருக்கங்களையோ பாவிக்காது,

(i) $p, q$ மற்றும் கோணம் $A\hat{D}C$ ஆகியவற்றைக் காண்க,

(ii) மேலும் $ABCD$ ஆனது ஒரு சாய்சதுரமல்ல எனவும் காட்டுக.

**2.** தரப்பட்ட இரு காவிகள் $\underline{a}$ மற்றும் $\underline{b}$ என்பனவற்றின் (i) எண்ணிப்பெருக்கம் $\underline{a}\cdot\underline{b}$ மற்றும் (ii) காவிப்பெருக்கம் $\underline{a}\times\underline{b}$ ஆகியவற்றினை வரையறுக்குக.

$A,B$ மற்றும் $C$ என்பன ஒரே கோட்டிலல்லாத தரப்பட்ட மூன்று புள்ளிகளும், $O$ ஆனது உற்பத்தியாகவுமுள்ளதெனின், $\left|\overline{OA}\times\overline{OB}\cdot\overline{OC}\right|$ என்பதன் பருமனை விபரிக்க.

உற்பத்தி $O$ குறித்த மூன்று புள்ளிகள் $A,B$ மற்றும் $C$ என்பனவற்றின் தானக்காவிகளான $\underline{a}, \underline{b}$ மற்றும் $\underline{c}$ என்பன முறையே $\underline{i}+2\underline{j}+3\underline{k}$, $2\underline{i}+\underline{j}+\underline{k}$ மற்றும் $\underline{i}-\underline{j}+2\underline{k}$ ஆகும்.

பின்வருவனவற்றைக் காண்க.

(i) கோணம் $A\hat{B}C$ இன் கோசைன்

(ii) முக்கோணி $ABC$ இன் பரப்பு

(iii) நான்முகி $OABC$ இன் கனவளவு.

6. ஒரு வெளி வளையியொன்றில் புள்ளி $P$ இன் தெக்காட்டின் ஆள்கூறுகள் $(x, y, z)$ என்பன பரமானம் $\theta$ சார்பாக, $x = c\cos\theta$, $y = c\sin\theta$, $z = (c\tan\alpha)\theta$ ஆல் தரப்படுகின்றன. இங்கு $c$ மற்றும் $\alpha$ என்பன $0 < \alpha < \dfrac{\pi}{2}$ ஆகுமாறுள்ள நேர் மாறிலிகளாகும்.

புள்ளி $(c, 0, 0)$ இலிருந்து அளக்கப்பட்ட $P$ இன் வில்லின் தூரமானது $s$ ஆல் குறிக்கப்படுகின்றது. $\dfrac{ds}{d\theta}$ ஐக் காண்க.

(i) அலகுத் தொடலிக் காவியானது $\hat{\mathbf{t}} = -\mathbf{i}\cos\alpha\sin\theta + \mathbf{j}\cos\alpha\cos\theta + \mathbf{k}\sin\alpha$ எனக் காட்டுக.

(ii) $\dfrac{d\hat{\mathbf{t}}}{ds} = \kappa\hat{\mathbf{n}}$ என்னும் சூத்திரத்தைப் பாவித்து, புள்ளி $P$ இல் வளைவு $\kappa$ ஐ $\dfrac{1}{c}\cos^2\alpha$ எனக் காட்டி, தலைமைச் செவ்வன் $\hat{\mathbf{n}}$ ஐக் காண்க.

(iii) இருமைச் செவ்வனானது $\hat{\mathbf{b}} = \mathbf{i}\sin\alpha\sin\theta - \mathbf{j}\sin\alpha\cos\theta + \mathbf{k}\cos\alpha$ எனக் காட்டுக.

(iv) $\dfrac{d\hat{\mathbf{b}}}{ds} = -\tau\hat{\mathbf{n}}$ என்னும் சூத்திரத்தைப் பாவித்து, $P$ இலுள்ள முறுக்கல் $\tau$ ஐக் காண்க.

மேற்படி வெளி வளைவிற்கு $\dfrac{d\hat{\mathbf{n}}}{ds} = \tau\hat{\mathbf{b}} - \kappa\hat{\mathbf{t}}$ ஐ வாய்ப்புப் பார்க்குக.