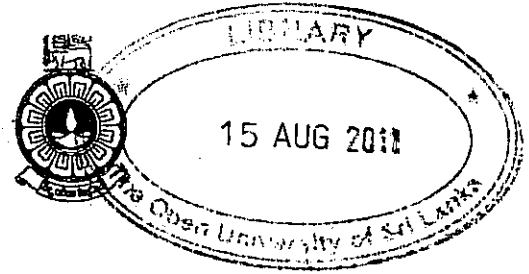


The Open University of Sri Lanka
 B.Sc./B.Ed Degree Programme – Level 04
 Final Examination 2010/2011
 Applied Mathematics
 APU 2143 – Vector Calculus
 Duration :- Two Hours.



Date :- 30.12.2010

Time:- 1.00 p.m. - 3.00p.m.

Answer Four Questions Only.

1. (a) Define the limit of a two variable function $f(x, y)$ as $(x, y) \rightarrow (a, b)$.
- (b) Define the continuity and differentiability of a two variable function at the point (a, b) .
- (c) Discuss the continuity and differentiability of the following functions.

$$(i) f(x, y) = \begin{cases} \frac{xy^2}{x^2 + y^4} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

$$(ii) f(x, y) = \begin{cases} xy \frac{x^2 - y^2}{x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0). \end{cases}$$

2. (a) If $\tan u = \frac{x^3 + y^3}{x - y}$, show that

$$(i) x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u \quad \text{and}$$

$$(ii) x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2 \cos 3u \sin u.$$

- (b) Using linear approximations, obtain an approximate value for the real number $\sqrt{9(1.95)^2 + (8.1)^2}$.
- (c) Find the directional derivative of $f(x, y, z) = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction $2\hat{i} - \hat{j} - 2\hat{k}$.

3. (a) (i) Define a stationary point of a single valued function $f(x, y)$ defined over the domain D . Explain briefly how you would determine its nature.

(ii) Find the maximum and minimum values of the function $f(x, y) = x^4 + y^4 - 4xy + 1$ and determine their nature.

- (b) Use method of Lagrange multipliers to minimize the function $f(x, y) = x^2 - xy + 2y^2$ subject to $2x + y = 22$.

4. (a) Find the value of the surface integral of the function $f(x, y) = x^2 + y^2$ over the closed region bounded by $y = x$ and $y^2 = 4x$.

- (b) Find the value of the surface integral of the function $f(x, y) = 4 - x^2 - y^2$ over the region in the first quadrant bounded by $x^2 + y^2 = 1$, $x^2 + y^2 = 9$, $y = 0$ and $x = 0$.

- (c) Find the volume integral of the function $f(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2}}$ over the hemispherical region bounded by the surfaces $x^2 + y^2 + z^2 = 3$ and $z = 0$.

5. (a) State Gauss' divergence theorem, giving the meanings of any symbols used.

- (b) Verify Gauss' divergence theorem for the vector field $\underline{F} = x^3 \underline{i} + y^3 \underline{j} + z^3 \underline{k}$, considering the region enclosed by the surface S of a sphere of radius R with centre at the origin.

- (c) Find the divergences of the following vector fields.

(i) \underline{r} ,

(ii) $r^n \underline{c}$ ($r \neq 0$) where $\underline{r} = x\underline{i} + y\underline{j} + z\underline{k}$, $r = |\underline{r}|$ and \underline{c} is a constant vector.

6. (a) Define what is meant by a conservative vector field.

Show that the vector field $\underline{F} = 2xy^2ze^{-x^2z}\underline{i} - 2ye^{-x^2z}\underline{j} + x^2y^2e^{-x^2z}\underline{k}$ is irrotational, and determine the corresponding scalar potential ϕ such that $\underline{F} = \nabla\phi$.

(b) Given two functions $P(x, y)$ and $Q(x, y)$ defined in \mathbb{R}^2 , prove Green's theorem

$$\oint_C (Pdx + Qdy) = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy.$$

(i) Through an appropriate choice of P and Q , obtain an expression for the area of the region R .

(ii) Hence find the area of the region enclosed by $y = 3x^2$ and $y = 6x$.