The Open University of Sri Lanka
B.Sc. Degree Programme
Open Book Test (OBT)-2009/2010
Pure Mathematics - Level 04
PMU2194/PME4194 — Number Theory & Polynomials

Duration: One and Half (1 ½) Hours

16 AUG 2011

Date: 22.03. 2010 **Time: 4.00 pm –5.30 pm**

ANSWER ALL QUESTIONS.

01. Let $A_{n+1}=(1-\alpha)(1-A_n)+A_n$ and $A_1=\beta$ for $n=1, 2, 3, \ldots$ where α and β are real numbers.

Prove by using the principle of Mathematical Induction that $A_n=1-(1-\beta)\alpha^{n-1}$ for every integer n.

Deduce $\sum_{r=1}^{n} A_r = n-(1-\beta)\dfrac{(1-\alpha^n)}{(1-\alpha)}$.

02. (a) Which of the following are inductive sets?

(i) $[-100,100^{100}]$ (ii) $(1, \infty)$ (iii) $(0, \infty)$ (iv) $[1, 10^{100}]$.

(b) Let $R[x]$ be a Commutative ring of polynomials. If $f(x), g(x) \in R[x]$ are non-zero polynomials such that $f(x)+g(x) \neq 0$ and $f(x)\cdot g(x) \neq 0$ then defining $R[x]$ clearly, give counter examples (not in your course material) to show that

(i) $\deg(f+g) \leq \max(\deg f, \deg g)$,
(ii) $\deg(f\cdot g) \leq \deg f+\deg g$.

03 (a) Suppose that polynomials $f(x)=x^4-1$ and $g(x)=x^3-1$ are in $\mathbb{Z}[x]$. Find the greatest common divisor of f and g.

(b) Use Division Algorithm to find the remainder and quotient when $x^4+2x^3+3x^2+x+1$ is divided by $x-3$ in $\mathbb{Z}_5[x]$.

(c) Solve the equation $x^4+3x^3+6x^2+2x+2=0$ in $\mathbb{Z}_7[x]$.

The Open University of Sri Lanka

B.Sc. Degree Programme- Level 04

Closed Book Test (CBT) 2009/2010

PMU 2194/PME 4194 - Number Theory & Polynomials

Duration :- One and Half Hours

LIBRARY 16 AUG 2011 The Open University

Date :- 23-04-2010 Time :- 4.00 p.m. – 5.30 p.m.

Answer All Questions.

(1) (i) Show that $\mathbb{Z} = \mathbb{N} \cup (-\mathbb{N}) \cup \{0\}$ where $\mathbb{Z}$ is the set of integers and $\mathbb{N}$ is the set of natural numbers.

(ii) If $z_1, z_2 \in \mathbb{Z}$ then prove the following,

(a) $z_1 + z_2 \in \mathbb{Z}$,

(b) $z_1 - z_2 \in \mathbb{Z}$,

(c) $z_1 \cdot z_2 \in \mathbb{Z}$.

(iii) If $a, b \in \mathbb{Z}$ and $ab = 1$ then prove that $|a| = 1$ and $|b| = 1$.

(Hint: if $n \in \mathbb{N}$ then $n \geq 1$)

(2) (a) Find the greatest common devisor of 4203 and 207. Express it in the form $4203m + 207n$ with suitable integers m and n. Find the least common multiple of 4081 and 319.

(b) (i) State Eisentein's irreducibility criteria.

Give Counter example to show that in Eisentein's irreducibility criteria, "f is irreducible in $\mathbb{Q}[x]$" cannot be replaced by "f is irreducible in $\mathbb{Z}[x]$".

(ii) Express $3x^4 + 4x^3 + 3x^2 + x + 1$ as a product of a unit and monic irreducible polynomials in $\mathbb{Z}_5[x]$.

(3) (a) Let $f(x)=\sum_{i=0}^{n} a_i x^i \in \mathbb{Z}[x]$ and $n \geq 1$. If $\alpha \in \mathbb{Q}$ is a zero of $f(x)$ and $\alpha=\frac{r}{s}$ with $(r, s)=1$, then $r \mid a_0$ and $s \mid a_n$.

Find rational roots of the polynomial $3x^4-40x^3+130x^2-120x+27$.

(b) Suppose that polynomials $f(x)=2x^3+2x^2+x+1$ and $g(x)=x^2+x+1$ are in $\mathbb{Z}_5[x]$. Find the greatest common divisor of f and g and express it in the form $d=fu+gv$ with $u, v \in \mathbb{Z}_3[x]$.