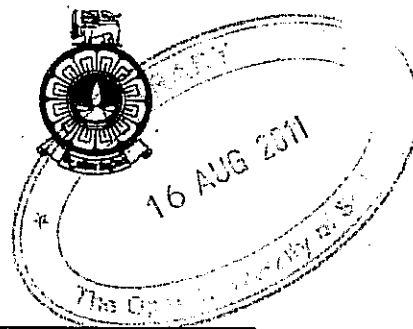


The Open University of Sri Lanka  
B.Sc. Degree Programme  
Open Book Test (OBT)-2009/2010  
Pure Mathematics - Level 04  
PMU2194/PME4194 — Number Theory & Polynomials



Duration: One and Half (1 ½) Hours

Date: 22.03. 2010

Time: 4.00 pm –5.30 pm

ANSWER ALL QUESTIONS.

01. Let  $A_{n+1} = (1-\alpha)(1-A_n) + A_n$  and  $A_1 = \beta$  for  $n=1, 2, 3, \dots$  where  $\alpha$  and  $\beta$  are real numbers.

Prove by using the principle of Mathematical Induction that  $A_n = 1 - (1-\beta)\alpha^{n-1}$  for every integer  $n$ .

Deduce 
$$\sum_{r=1}^n A_r = n - (1-\beta) \frac{(1-\alpha^n)}{(1-\alpha)}.$$

02. (a) Which of the following are inductive sets?

- (i)  $[-100, 100^{100}]$     (ii)  $(1, \infty)$     (iii)  $(0, \infty)$     (iv)  $[1, 10^{100}]$ .

(b) Let  $R[x]$  be a Commutative ring of polynomials. If  $f(x), g(x) \in R[x]$  are non-zero polynomials such that  $f(x) + g(x) \neq 0$  and  $f(x) \cdot g(x) \neq 0$  then defining  $R[x]$  clearly, give counter examples (not in your course material) to show that

- (i)  $\deg(f + g) \leq \max(\deg f, \deg g)$ ,  
(ii)  $\deg(f \cdot g) \leq \deg f + \deg g$ .

03 (a) Suppose that polynomials  $f(x) = x^4 - 1$  and  $g(x) = x^3 - 1$  are in  $\mathbb{Z}[x]$ . Find the greatest common divisor of  $f$  and  $g$ .

(b) Use Division Algorithm to find the remainder and quotient when  $x^4 + 2x^3 + 3x^2 + x + 1$  is divided by  $x - 3$  in  $\mathbb{Z}_5[x]$ .

(c) Solve the equation  $x^4 + 3x^3 + 6x^2 + 2x + 2 = 0$  in  $\mathbb{Z}_7[x]$ .

The Open University of Sri Lanka

B.Sc. Degree Programme- Level 04

Closed Book Test (CBT) 2009/2010

PMU 2194/PME 4194 - Number Theory & Polynomials

Duration :- One and Half Hours



Date :- 23-04-2010

Time :- 4.00 p.m. – 5.30 p.m.

Answer All Questions.

(1) (i) Show that  $\mathbb{Z} = \mathbb{N} \cup (-\mathbb{N}) \cup \{0\}$  where  $\mathbb{Z}$  is the set of integers and  $\mathbb{N}$  is the set of natural numbers.

(ii) If  $z_1, z_2 \in \mathbb{Z}$  then prove the following,

(a)  $z_1 + z_2 \in \mathbb{Z}$ ,

(b)  $z_1 - z_2 \in \mathbb{Z}$ ,

(c)  $z_1 \cdot z_2 \in \mathbb{Z}$ .

(iii) If  $a, b \in \mathbb{Z}$  and  $ab = 1$  then prove that  $|a| = 1$  and  $|b| = 1$ .

(Hint: if  $n \in \mathbb{N}$  then  $n \geq 1$ )

(2) (a) Find the greatest common divisor of 4203 and 207. Express it in the form  $4203m + 207n$  with suitable integers  $m$  and  $n$ . Find the least common multiple of 4081 and 319.

(b) (i) State Eisenstein's irreducibility criteria.

Give Counter example to show that in Eisenstein's irreducibility criteria, "f is irreducible in  $\mathbb{Q}[x]$ " cannot be replaced by "f is irreducible in  $\mathbb{Z}[x]$ ".

(ii) Express  $3x^4 + 4x^3 + 3x^2 + x + 1$  as a product of a unit and monic irreducible polynomials in  $\mathbb{Z}_5[x]$ .

- (3) (a) Let  $f(x) = \sum_{i=0}^n a_i x^i \in \mathbb{Z}[x]$  and  $n \geq 1$ . If  $\alpha \in \mathbb{Q}$  is a zero of  $f(x)$  and  $\alpha = \frac{r}{s}$  with  $(r, s) = 1$ , then  $r \mid a_0$  and  $s \mid a_n$ .

Find rational roots of the polynomial  $3x^4 - 40x^3 + 130x^2 - 120x + 27$ .

- (b) Suppose that polynomials  $f(x) = 2x^3 + 2x^2 + x + 1$  and  $g(x) = x^2 + x + 1$  are in  $\mathbb{Z}_5[x]$ . Find the greatest common divisor of  $f$  and  $g$  and express it in the form  $d = fu + gv$  with  $u, v \in \mathbb{Z}_5[x]$ .