The Open University of Sri Lanka
B.Sc. /B.Ed. Degree Programme
Final Examination – 2010/2011
Level 05-Applied Mathematics
AMU 3183/AME 5183 – Numerical Methods II

Duration: - Two Hours

Date: - 17-12-2010. **Time: - 1.30 p.m. – 3.30 p.m.**

Answer Four Questions Only.

1. (a) Derive the equations to fit straight line model to the data

$(x_1, y_1), (x_2, y_2), \dots, (x_n, y_n)$ by method of least square.

(b) The height H and the quantity Q of water flowing per second are related by the law $Q = CH^n$, where C and n are constants. The quantity of water Q for seven different height H are presented in the accompanying table.

$H(ft)$	1.2	1.4	1.6	1.8	2.0	2.2	2.4
$Q(ft)^3$	4.2	6.7	8.5	11.5	14.9	23.5	27.1

(i) Using least squares fitting, estimate the values of C and n?

(ii) Write down the model. Hence estimate the value of Q corresponding to H=3ft.

2. (a) Derive Euler's method to find solutions of the first order differential equation $\frac{dy}{dx} = f(x, y)$ under the initial condition $y(x_0) = y_0$.

(b) Hence, solve the differential equation $\frac{dy}{dx} - x^2 = y^2$, subjected to the initial condition $y(0) = 1$. Assume h=0.05. Round your results for the appropriate number of decimal places at x=0.05 and x= 0.10.

3. (a) With the usual notation prove the Simpson's rule.

(b) Prove that the truncation error E in using Trapezoidal rule for the integral

$\int_a^b f(x)\,dx$ is given by $E = \frac{-(b-a)h^2}{12} f''(c)$; where $c \in (a,b)$ and $h = \frac{b-a}{n}$.

(c) A car laps a race track in 84 seconds. The speed of the car at each 6 seconds interval is determined using a radar gun and given from the beginning of the lap in *m/s*. The records are as follows:

Time(s)	0	6	12	18	24	30	36	42	48	54	60	66	72	78	84
Speed(m/s)	124	134	148	156	147	133	121	109	99	85	78	89	104	116	123

Find the track length by using

(i) Trapezoidal rule (ii) Simpson's rule

4. (a) Write down to fourth order Runge –Kutta method to find an approximate solution to the differential equation $\frac{dy}{dx} = f(x,y)$ subject to the initial condition $y(x_0) = y_0$.

(b) Use fourth order Runge –Kutta method to solve the differential equation $\frac{dy}{dx} = \frac{y^2+2x}{y^2+x}$, subject to the initial condition $y(0)=1$ with $h=0.1$. Hence find the value of y at $x = 0.1,\ 0.2,\ 0.3$. Use the order of the error term to round your results for the appropriate number of decimal places.

5. (a) Use Newton forward difference formulae to obtain the value of $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at $x = 1.3$.

The values of x and y given below.

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

(b) Construct the difference table to check the following sequence of data and find the correct error in the list.

1, 2, 4, 8, 16, 26, 42, 64, 93.

6. (a) Write down the n^{th} order Lagrange's interpolation polynomial *P(x)* for the data $(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$.

(b) With the usual notation, prove that the error of interpolation of Lagrange's method is

$\frac{\pi(x)}{(n+1)!} f^{n+1}(c)$, where $\pi(x) = (x-x_0)\times(x-x_1)\times\dots\times(x-x_n)$ and c is in the smallest interval contains $x_0, x_1, \dots, x_n$.

(c) Find the second degree Lagrange's polynomial $y = p(x)$ for the curve $y(x) = \sin\frac{\pi x}{2}$ which takes the following values.

x_k	0	1	2
y_k	0	1	0

(d) Using part (b) and, show that $y(x) - p(x) = -\frac{\pi^2}{48}\cos\frac{\pi\mu}{2} x(x-1)(x-2)$, where μ depend on *x*.