16 AUG 2011

The Open University of Sri Lanka
B.Sc./B/Ed. Degree Programme-2010/2011
Closed Book Test (CBT)
Pure Mathematics
PUU2144-Group Theory I

Duration: One and Half Hours

Date: 18.10.2010 Time: 4.00pm-5.30pm

Answer All Questions

1. (a) In S_4, find the subgroup H generated by $(1\ \ 2\ \ 3)$ and $(1\ \ 2)$.
For the subgroup H of S_4 find the corresponding subgroup $\sigma H\sigma^{-1}$ for $\sigma=(1\ \ 4)$.

(b) Show that each element in A_4 can be written as a product of 3- cycles.

2. State and prove the Lagrange's theorem.
Find all the subgroups of $\mathbb{Z}_{12}=\{1,a,a^2,\ldots,a^{11}\}$.
Determine whether $\mathbb{Z}_8$ has a subgroup of order 5.

3. (a) Show that each subgroup of an abelian group is normal.

(b) If $H\leq G$ such that $|G:H|=2$ then $H\trianglelefteq G$.