



The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme- Level 4
Open Book Test (OBT)- 2010/2011
Pure Mathematics
Group Theory I - PUU2144

Duration: One and Half hours

Date: 10. 09. 2010

Time: 4.00p.m.-5.30pm.

Answer all the Questions.

1. (a) On the set $G = \mathcal{Q}^*$ of non-zero rational number, define a new multiplication by

$$a * b = \frac{ab}{2} \text{ for all } a, b \in G.$$

Show that G is a group under this multiplication.

(b) Write out the multiplication table for Z_9^* .

2. (a) Let G be a group and suppose that a and b are any elements of G .

Show that $(aba^{-1})^n = ab^n a^{-1}$ for any positive integer n .

(b) Let G be an abelian group and let n be a fixed positive integer.

Show that $N = \{g \in G \mid g = a^n \text{ for some } a \in G\}$ is a subgroup of G .

3. Let $A = \{1, 2, 3\}$ be a set. Write down all the permutations.

Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 2 & 3 & 1 & 4 & 6 & 5 & 7 & 8 & 10 & 9 \end{pmatrix}$ be a permutation.

Write down σ in the cyclic notation. State whether that σ is an even or odd permutation. Justify your answer.

If $\alpha = (1 \ 4 \ 5)(2 \ 3)$ and $\rho = (2 \ 4)(5 \ 1)$, then find $\alpha\rho$ and α^2 .

Let $\beta = (1 \ 3 \ 6 \ 2)(3 \ 7)(2 \ 4 \ 5)$. Find the inverse of β .