The Open University of Sri Lanka
B.Sc./B/Ed. Degree Programme-2010/2011
Closed Book Test (CBT)
Pure Mathematics
PUU2140-Sequences and Series

Duration: One and Half Hours

Date: 28.10.2010 Time: 4.00pm-5.30pm

Answer All Questions

1. (a) State and prove the Sandwich Theorem.

Use the Sandwich Theorem to show that the sequence $\langle a_n \rangle$ where $a_n = \frac{\sin(n)}{n} \to 0$ as $n \to \infty$.

(b) State the ε - definition of a divergent sequence.

Show that the sequence $\langle a_n \rangle$ where $a_n = \frac{n^2+5n}{3n+2}$ is divergent.

2. Show that the sequence $\langle x_n \rangle$ where $\sum_{k=1}^{n} \frac{1}{k^2}$ is a Cauchy sequence.

3. (a) Find the limits of the following series.

(i) $\sum_{n=1}^{\infty} \left(\frac{1}{2^n} + \frac{1}{3^n} \right)$ (ii) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

(iii) $\sum_{n=1}^{\infty} \frac{1}{n}$

(b) Use the comparison test to show that the following series are convergent.

(i) $\sum_{n=1}^{\infty} \frac{2n}{3n^3-1}$ (ii) $\sum_{n=1}^{\infty} \frac{\log n}{n}$