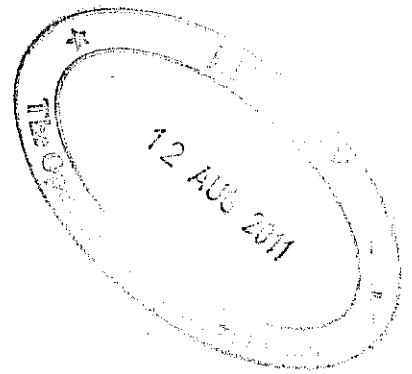


The Open University of Sri Lanka  
 B.Sc./B.Ed. Degree Programme-Level 04  
 Open Book Test (OBT)- 2010/2011  
 Pure Mathematics



PUU2140-Sequences and Series

**Duration: One and Half Hours**

Date: 13.09.2010

Time: 4.00 pm.-5.30 pm.

**Answer All Questions**

1. (a) Let  $a_n = \left(1 + \frac{1}{n}\right)^{n+1}$ ;  $n \geq 1$  be the  $n^{\text{th}}$  term of a sequence. Show that  $\langle a_n \rangle$  is a decreasing sequence.

(b) Is it true that a bounded sequence converges? Justify your answer.  
 Is it true that a monotone increasing sequence converges? Justify your answer.

2. (a) Using the definition of  $\lim_{n \rightarrow \infty}$ , show that  $\lim_{n \rightarrow \infty} \frac{4n^2 + 8}{2n^2 + n} = 2$ .

(b) Prove that  $\langle (-1)^n \rangle$  does not converge.

3. (a) Let  $a_1 = \sqrt{2}$  and let  $a_{n+1} = \sqrt{2a_n}$  for  $n \geq 1$ . Show that  $\langle a_n \rangle$  is increasing and bounded.

(b) Prove that  $\lim_{n \rightarrow \infty} \frac{1}{n} = 0$  using  $\varepsilon$ -definition of limit.

Deduce that  $\lim_{n \rightarrow \infty} \frac{n^5 + 3n + 1}{3n^5 + 2n + 5} = \frac{1}{3}$ .