THE OPEN UNIVERSITY OF SRI LANKA
DIPLOMA IN TECHNOLOGY /BACHELOR OF SOFTWARE ENGINEERING – LEVEL 04
FINAL EXAMINATION– 2012/2013
MPZ4140/MPZ4160– DISCRETE MATHEMATICS - I
DURATION – THREE (03) HOURS

Date: 29th July 2013 Time: 1330-1630 hours

Instructions:

- **Answer only six questions.**
- **Please answer a total of six questions choosing at least one from each section.**
- **State any assumptions that you made.**
- **All symbols are in standard notation.**

SECTION – A

01. i. Define a "Statement" and "Compound statement".

(10 marks)

ii. Determine the truth value (True or False) of the following proposition

a) If the earth is round then the earth travels round the sun.
b) If dog has 5 legs, then rat has two legs.
c) Pasindu and Menuka are brothers if and only if Pasindu and Menuka share the same family name.

(15 marks)

iii. Let p, q, r be three statements. Verify that each the following statement is whether tautology or not.

a) $(p \to q) \leftrightarrow [(p \vee r) \to (q \vee r)]$
b) $[(p \to q) \wedge (r \to q)] \leftrightarrow [(p \vee r) \to q]$

(50 marks)

iv. Use the laws of the algebra of propositions to show that,
$\sim (p \vee q) \vee (\sim p \wedge q) \equiv \sim p$

(25 marks)

02. i. Determine the contrapositive of each the following statement:

a) If he has courage he will win,

b) It is sufficient for it to be a square in order to be a rectangle.

(10 marks)

ii. Express each of the following statements in words, state whether it is true or false with justification.

a) $\forall x \in \mathbb{R},\ x^2 - 4x + 3 > 0$

b) $\exists n \in \mathbb{N},\ 41\ and\ 43$ are divisors of $n^2 + n$.

(30 marks)

iii. Test the validity of the following argument. "If interest rates are law, then housing starts are up. If housing starts are up, then marriage rates are high. If interest rates are low, then the economy is good. The economy is not good. Therefore marriage rate are not high.

(40 marks)

iv. Disprove the following statements:

a) For each positive integer n, $\sqrt{n} = n$,

b) all odd numbers are divisible by 3.

(20 marks)

03. i. Prove that, if $x + y$ is an irrational number, then x is an irrational number or y is an irrational number.

(25 marks)

ii. Using mathematical induction, prove that, for each $n \in \mathbb{Z}^+$.

a) The sum of the first n odd numbers is n^2.

b) $\dfrac{x^{n+1} - 1}{x - 1} = 1 + x + x^2 + ... + x^n$ where $x \neq 1$ *and* $x \in \mathbb{R}$.

(50 marks)

iii. Show that $\sqrt{2}$ is an irrational number.

(25 marks)

SECTION – B

04. i. Which of the following set is the null set?

a) $A = \{x \mid x$ is a letter before a in the alphabet$\}$,

b) $B = \{x \mid x + 3 = 3,\ x \in \mathbb{Z}\}$

c) $C = \{x \mid x^2 = 16 \text{ and } 4x = 8,\ x \in \mathbb{Z}\}$

(15 marks)

ii. Let $D = \{0, 1, \{3, 4\}\}$. Find all the subsets of D.

(10 marks)

iii. Using algebraic laws of sets, show that $(A \cap B)' = A' \cup B'$.

(35 marks)

iv. Let $E = \{x \in \mathbb{N} \mid x < 10\}$,

$F = \{x \in \mathbb{Z} \mid |x - 3| < 5\}$, and

$G = \{x \in \mathbb{R} \mid x^3 - 9x = 0\}$

a) List the elements of each of these above set.

b) With usual notation, find $E \cup G,\ F \cap G,\ E \setminus F,\ E \oplus F$ and $F \setminus (F \oplus G)$

(40 marks)

05. i. Let R be the relation on the natural numbers $\mathbb{N}$ defined by,

$R = \{(x, y) \mid x \in \mathbb{N},\ y \in \mathbb{N},\ x + 3y = 12\}$.

a) Write R as a set of ordered pairs.

b) Find the domain of R and R^{-1}.

(15 marks)

ii. Each of the following sentences defines a relation on the natural numbers $\mathbb{N}$.

R_1 ; "x is a multiple of y"

R_2 ; "x + 3y = 12"

R_3 ; "x is disjoint from y"

State whether or not each of the relations is

a) reflexive

b) symmetric

c) anti-symmetric

(20 marks)

iii. What is meant by equivalence relation on set A.
Show that the relation,
$R = \{(a,b) \mid (a-b) = kn \text{ for some fixed integer } n \text{ and } a,b,k \in \mathbb{Z}\}$ is an equivalence relation.

(35 marks)

iv. Show that "$x \leq y$" on the set of integers is a Partial order.

(30 marks)

06. i. Explain the main difference between an ordered pair (a,b) and the set $\{a,b\}$.

(10 marks)

ii. Let $A = \{a,b\},\ b = \{1,2,3,4,5,6\},\ \text{and}\ C = \{3,5,7,9\}$. Find $(A \times B) \cap (A \times C)$.

(15 marks)

iii. Define a composition function.
Let $F : \mathbb{R} \to \mathbb{R}$ be a function and defined as $f(x) = x^2$ and $g : \mathbb{R} \to \mathbb{R}$ is a function and defined as

$$g(x) = \begin{cases} x-3 & ; \quad x < 0 \\ 2x & ; \quad x \geq 0 \end{cases}$$

Obtain the composition functions *fog* and *gof*.

(40 marks)

iv. Let the function $h : \mathbb{R} \to \mathbb{R}$ be defined by

$$h(x) = \begin{cases} 2x+5 & ; \quad x > 9 \\ x^2 - |x| & ; \quad x \in [-9,9] \\ x-4 & ; \quad x < -9 \end{cases}$$

Find $h(x)$ and $h(h(5))$.

(15marks)

v. Prove that $f : \mathbb{N} \to \mathbb{N}$ where $f(x) = x^2$ is one to one, but function $g : \mathbb{Z} \to \mathbb{Z}$ where $g(x) = x^2$ is not one to one.

(20marks)

SECTION – C

07. i. Show that,

a) $\gcd(a,b) = \gcd(a, b-2a) = \gcd(a, b-3a)$,

b) if $a|b$ and $b|a$, then $a \pm b$,

c) if $\gcd(a,b)=1$ and $\gcd(a,c)=1$, then $\gcd(a,bc)=1$.

(75 marks)

ii. a) Define a Prime number.

b) If $n>1$, $n \in \mathbb{N}$, then show that there is a Prime number P such that $P|n$.

(25marks)

08. i. Show that $\gcd(a,b)lcm(a,b) = ab$, for positive integers a and b.

(25marks)

ii. If $\gcd(a,b)=1$, then show that $\gcd(a+b,\ a-b) = 1\ or\ 2$.

(25marks)

iii. Use the Euclidean Algorithm to find the integers x and y of the equation:

$$\gcd(172,\ 20) = 172x + 20y.$$

Determine all integer solutions of the following Diophantine equation.
$172x + 20y = 1000$.

(50marks)

09. i. Let $m>0,\ m \in \mathbb{R}$, be a fixed number and a,b,c,d be four arbitrary integers.

a) If $a \equiv b (\bmod m)$ and $c \equiv d (\bmod m)$, then prove that $ac \equiv bd (\bmod m)$.

b) If $a \equiv b (\bmod m)$, then prove that $a^k = b^k (\bmod m)$ for any positive integer k.

c) If $a \equiv b (\bmod m)$ and $c \equiv d (\bmod m)$, then prove that $2a - 3c \equiv (2b - 3d)(\bmod m)$.

(60marks)

ii. Solve the following set of congruences simultaneously.
$x \equiv 5 (\bmod 11)$,
$x \equiv 14 (\bmod 29)$,
$x \equiv 15 (\bmod 31)$.

(40marks)