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THE OPEN UNIVERSITY OF SRI LANKA
B.Sc./B.Ed. Degree Programme, Continuing Education Programme
APPLIED MATHEMATICS – LEVEL 05
AMU 3189/ AME 5189- STATISTICS II
FINAL EXAMINATION - 2009/2010

Duration: Two and Half Hours.

Date: 30.06.2010	**Time: 9.30 a.m. – 12.00 noon**

Non programmable calculators are permitted. Statistical tables are provided.

Answer FOUR questions only.

(1) The random variable X denotes the lifetime of a certain type of battery. The probability of X is given by $f(x;\lambda)=\frac{x}{\lambda^2}e^{-\frac{x}{\lambda}}$ and the moment generating function of X is given by $M_x(t)=(1-\lambda t)^{-2}$. Let $X_1, X_2, \ldots X_n$ denote lifetimes of n randomly chosen batteries from the above population.

(a) Find the mean and variance of X

(b) Find the maximum likelihood estimator of λ.

(c) Find the Cramer-Rao lower bound (CRLB) for the variance of an unbiased estimator of λ

(d) Is the estimator found in part (b) above a uniformly minimum variance unbiased estimator (UMVUE) for λ? Give reasons for your answer.

(2) Let $X_1, X_2, \ldots X_n$ be a random sample from a Normal distribution with mean μ and variance σ^2 so that the probability density function of X_i is given by

$$f(x_i;\mu,\sigma)=\frac{1}{\sqrt{2\pi\sigma}}\exp\left\{-\frac{(x_i-\mu)^2}{2\sigma^2}\right\} \quad ; \quad -\infty < x_i < \infty$$

(a) Show that the sample variance $s^2=\frac{\sum_{i=1}^{n}(Xi-\overline{X})^2}{n-1}$ is an unbiased estimator for σ^2.

(b) Using the Factorization Theorem show that $\sum_{i=1}^{n} X_i$ and $\sum_{i=1}^{n} X_i^2$ are jointly sufficient for μ and σ^2.

(c) Is sample mean $\overline{X}$ and sample variance s^2 jointly sufficient for μ and σ^2? Justify your answer.

(d) Is $\sum_{i=1}^{n} X_i$ and $\left(\sum_{i=1}^{n} X_i^2\right)^2$ jointly sufficient for μ and σ^2? Justify your answer.

(3) An inventor has developed a new, energy-efficient lawn mower engine. He claims that the engine will run continuously for 5 hours (300 minutes) on a single gallon of regular gasoline. To test inventor's claim a simple random sample of 25 engines are tested. The engines run for an average of 295 minutes, with a standard deviation of 20 minutes. Assume that run times for the population of engines are normally distributed.

(a) Construct 95% confidence interval for the mean runtime of the engine on a single gallon of regular gasoline. Comment about the inventor's claim.

(b) Suppose you are asked to conduct a statistical test to test the inventor's claim

(i) Clearly sate the null hypothesis and the alternative hypothesis that you should test.

(ii) Find Likelihood ratio test statistic and critical region to test the hypothesis mention in (i)

(iii) What is your conclusion at 0.5 level of significance?

(4) An investigation was conducted in to the dust content in the flue gases of two types of solid – fuel boilers. Thirteen boilers of type A and nine boilers of type B were used under identical fuelling and extraction conditions. Over a similar period, the following quantities, in grams, of dust were deposited in similar traps inserted in each of the twenty- two flues.

Type A	73.1	56.4	82.1	67.2	78.7	75.1	48.0
	53.3	55.5	61.5	60.6	55.2	63.1	
Type B	53.0	39.3	55.8	58.8	41.2	66.6	46.0
	56.4	58.9					

Assume that these independent samples come from normal populations. Sample means of dust contents of type A and type B are 63.83 grams and 52.89 grams respectively. Sample standard deviations of the dust contents of type A and type B are 10.63 grams and 9.00 grams respectively.

(a) Test for an equality of population variances. Use 0.1 level of significance.

(b) Test for an equality of population means. Use 0.1 level of significance.

(c) Do the dust contents in the flue gases of two types of solid – fuel boilers have same distribution? Justify your answer.

(5)

(a) Mannan, R.W., and E.C. Meslow. (*1984, Bird populations and vegetation characteristics in managed and old-growth forests, northeastern Oregon. J. Wildl. Manage. 48: 1219-1238*) studied bird foraging behavior in a forest in Oregon. In a managed forest, 54% of the canopy volume was Douglas fir, 40% was ponderosa pine, 5% was grand fir, and 1% was western larch. They made 156 observations of foraging; 70 observations (44.9% of the total) in Douglas fir, 79 (50.6% of the total) in ponderosa pine, 3 (2% of the total) in grand fir, and 4 (2.5% of the total) in western larch.

The biological null hypothesis is that the birds forage randomly, without regard to what species of tree they're in; the statistical null hypothesis is that the proportions of foraging events are equal to the proportions of canopy volume.

Conduct a statistical test and give your conclusion at 5% level of significance.

(b) In a study of the television viewing habits of children, a developmental psychologist selects a random sample of 300 first graders - 100 boys and 200 girls. Each child is asked which of the following TV programs they like best: The Punchi weerayo, Super man, or Cynderella. Results are shown in the contingency table below.

	Viewing Preferences			
	Punchi weerayo	Superman	Cynderella	Row total
Boys	30	65	05	100
Girls	60	40	100	200
Column total	90	105	105	300

Conduct a statistical test to test whether the Viewing Preferences depend on the gender. Clearly sate your null hypothesis and alternating hypothesis. Use 0.05 level of significance.

(6) A manager wishes to determine whether the mean times required to complete a certain task differ for the three levels of employee training. He randomly selected 5 employees with each of the three levels of training (Beginner, Intermediate and Advanced). The employees were assigned the task and the time required to complete the task was measured in minutes. Observed data are shown bellow.

Level of training	Advanced	20	10	60	70	60
	Intermediate	100	100	110	60	100
	Beginner	110	120	120	90	100

(a) What is the design structure used in this experiment? Justify.

(b) How many treatments are there and what are they?

(c) State the null hypothesis and the alternative hypothesis that should be tested to test whether the levels of training are equally effective. .

(d) Construct an analysis of variance (ANOVA) table.

(e) Test whether the levels of training are equally effective. Use 5% level of significance. Clearly state your findings.