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The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination – 2009/2010
Level 05-Applied Mathematics
AMU 3185/AME 5185 – EM Theory & Special Relativity
Duration :- Two and Half Hours

Date :- 14-07-2010. **Time :- ~~9.30 p.m. – 12.00 pm.~~** 1.00 p.m. – 3.30 p.m.

Answer Four Questions Only.

01. An imaginary open surface S is in the form of a spherical cap $r = a,\ 0 \le \theta \le \alpha$, $0 \le \omega \le 2\pi$, where (r, θ, ω) denote spherical polar coordinates. Define the flux of a vector $\underline{E}$ through S.

   (a) Use your definition to calculate the electric flux through S if a uniform electric field $\underline{E}$ acts parallel to the axis of symmetry?

   (b) What is the flux if $\underline{E}$ acts perpendicular to the axis of symmetry?

   (c) Determine the flux through S if a point charge Q is placed at the centre $(r = 0)$. Deduce the flux for the particular case $\alpha = \pi$.

02. Show that, if $\underline{A}$ satisfies the equations

$$div\,\underline{A} = 0, \qquad \nabla^2 \underline{A} = \frac{1}{c^2}\ddot{\underline{A}}$$

and $\underline{E}$ and $\underline{H}$ are defined by the relations

$$\underline{E} = -\frac{1}{c}\dot{\underline{A}}, \qquad \underline{H} = curl\,\underline{A},$$

then $\underline{E}$ and $\underline{H}$ satisfy the Maxwell's equations,

$$curl\,\underline{H} - \frac{1}{c}\dot{\underline{E}} = 0, \qquad div\,\underline{H} = 0$$

$$curl\,\underline{E} + \frac{1}{c}\dot{\underline{H}} = 0, \qquad div\,\underline{E} = 0$$

for the electromagnetic field in vacuuo. (Dots denote partial differentiation with respect to time)

Show that $\underline{A} = \underline{i}\,a\cos\frac{2\pi}{\lambda}(z - ct) + \underline{j}\,a\sin\frac{2\pi}{\lambda}(z - ct)$, where a and λ are constants, is a possible solution.

03. (a) Define the following terms.

(i) Resistivity (ii) Flux density

(ii) Charge density (iv) Potential

(b) A uniform volume charge distribution of -10^{-8} coulomb/(metre)^3 occupies the region between two co-axial conducting cylinders of radii 20 *mm* and 50 *mm*. If the electric field and potential are both zero on the inner cylinder, find the potential on the outer cylinder.

(Hint: Use the Poisson's equation.)

04. A conducting sphere *S* of *a* radius has a charge *Q*. Verify, by direct evaluation of the integral $\phi = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma}{r} ds$, that the potential is $\frac{Q}{4\pi\varepsilon_0 r}$ at points outside the sphere and $\frac{Q}{4\pi\varepsilon_0 a}$ at points outside *S*, *r* being the distance from the centre of the sphere.

Determine the electric field at any point and verify by performing the integral $W = \frac{\varepsilon_0}{2}\int \underline{E}^2 dW$, that the energy of the given conducting sphere $\frac{Q^2}{8\pi\varepsilon_0 a}$.

05. (a) Derive an expression for the magnetic field at any point on the line passing through the centre and perpendicular to the plane of a circular loop, which carries a current I.

(b) Derive an expression for the magnetic field at a point on the axis of a solenoid of radius R and N turns/metre, which carries a current I.

06. Derive the Lorezntz transformation equations.

Verify that the above equations can be expressed in the form

$$x' = x\cosh\alpha - ct\sinh\alpha$$
$$y' = y$$
$$z' = z$$
$$ct' = ct\cosh\alpha - x\sinh\alpha$$

where $\tanh\alpha = v/c$

Deduce that

$$x' - ct' = (x - ct)e^{\alpha}$$
$$x' + ct' = (x + ct)e^{\alpha}$$