The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination – 2009/2010
Level 05-Applied Mathematics
AMU 3181/AME 5181 – Fluid Mechanics
Duration :- Two and Half Hours

Date :- 21-06-2010. **Time :- 1.30 p.m. – 4.00 pm.**

Answer Four Questions Only.

01. In usual notation, show that the pressure gradient due to gravity $\frac{dp}{dz}$ is given by,

$$\frac{dp}{dz} = -\rho g.$$

The temperature in the Earth's atmosphere, at rest, remains constant and the pressure varies with the density according to Boyle's law $p = K\rho$, where K is a constant. The acceleration due to gravity at a height Z above the ground level is given by $g = \frac{g_0 a^2}{(a+z)^2}$, where a is the radius of the earth and g_0 is the value of g at ground level.

Show that the pressure P at a height Z is given by $P = P_0 e^{\frac{-g_0 a z}{K(a+z)}}$, where P_0 is the value of P at ground level.

02. Velocity, in spherical polar coordinates (r, θ, φ), at the point in a fluid is given by $\underline{q} = \left[-U\left(1 - \frac{a^3}{r^3}\right)\cos\theta,\ U\left(1 + \frac{a^3}{2r^3}\right)\sin\theta,\ 0\right]$. Show that this represents a possible motion, and determine the streamlines.

Verify that the motion is irrotational. Find the stagnation points and the velocity potential.

03. Two equal 2–D sources, each of strength m, are placed at points A $(a,0)$, B $(-a,0)$, in unbounded incompressible fluid. Write down the velocity potential and stream function.

Show that

(i) the streamlines are given by $x^2 - y^2 + \lambda xy = a^2$, where λ is a parameter.
(ii) the x–axis is a streamline, and find the velocity at any point there.
(iii) at any point on the circle with AB as a diameter, fluid velocity is parallel to the y- axis and its magnitude is inversely proportional to $|y|$.

00073

04. With the usual notation, assuming the Euler's equation

$$\frac{\partial U}{\partial t}+\nabla\left(\frac{1}{2}U^2\right)-U\wedge(\nabla\wedge U)=F-\frac{1}{\rho}\nabla P,$$

show that $\frac{P}{\rho}+\frac{1}{2}U^2-\Omega=$ Constant along the stream lines. State assumptions you make, if any.

A stream in a horizontal pipe, after passing a contraction in the pipe at which the sectional area is A, is delivered at atmospheric pressure at a place where the sectional area is B. Show that if a side tube is connected with pipe at the former place, water will be sucked up through it into the pipe from the reservoir at a depth, $\frac{S^2}{2g}\left(\frac{1}{A^2}-\frac{1}{B^2}\right)$, below the pipe, where S is the delivery per second and g is the gravity.

05. Find the components in the two–dimensional motion represented by the stream function $\psi=U\left[y-a\tan^{-1}(y/x)\right]$, verify that the motion is irrotational, and find the velocity potential. Show that the velocity at infinity is U, in the negative Ox–direction.

06. The velocity vector $\underline{q}$ in a motion of a perfect incompressible fluid of density ρ is given as

$$\underline{q}=\begin{cases}\omega r\underline{e}_{\theta}, & 0\le r<a\\ \dfrac{\omega a^2}{r}\underline{e}_{\theta}, & r\ge a,\end{cases}$$

Referring to cylindrical polar coordinates (r,θ,z), where ω is a positive constant, find the **vorticity vector** and identify the type of motion, in each region.
If the fluid motion takes place under no external forces and pressure at infinity is p_{∞}, use Euler's equation of motion to find the pressure distribution in the (outer) region $r>a$. Show that the pressure in the inner region is $p_{\infty}+\frac{\rho}{2}\omega^2\left(r^2-2a^2\right)$.