THE OPEN UNIVERSITY OF SRI LANKA
BACHELOR OF SOFTWARE ENGINEERING
DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

ECZ3161 – MATHEMATICS FOR COMPUTING
FINAL EXAMINATION – 2013/14 **CLOSED BOOK**

Date: August 6, 2014 **Time: 09.30-12.30 hrs**

Instructions

1. Answer any **five** out of eight questions. All question carry equal marks.
2. Show all steps clearly.
3. **Programmable** calculators are **not** allowed.

Q1

(a) Use Boolean algebra to simplify following expressions.

i) $\bar{a}\bar{b}\bar{c} + \bar{a}\bar{b}c + ab\bar{c} + abc = ab + \bar{a}\bar{b}$

ii) $\overline{\bar{a}\bar{b}\bar{c} + abc + \bar{a}\bar{b}c + a\bar{b}c} = \bar{a}c + a\bar{c}$

(b) Use Truth tables to show the followings.

i) $xyz + xy\bar{z} + x\bar{y}z + \bar{x}yz + \bar{x}\bar{y}z = xy + z$

ii) $\overline{xyz + xy\bar{z} + \bar{x}y\bar{z} + \bar{x}\bar{y}z} = x\bar{y} + \bar{y}\bar{z} + \bar{x}yz$

(c) Use Karnaugh map and find minimal sum for the followings.

i) $xy\bar{z} + \bar{x}\bar{y}\bar{z} + x\bar{y}\bar{z} + \bar{x}y\bar{z}$

ii) $x\bar{y}z\bar{t} + x\bar{y}\bar{z}\bar{t} + \bar{x}\bar{y}\bar{z}\bar{t} + \bar{x}\bar{y}z\bar{t}$

Q2

(a)

If $A = \begin{pmatrix} 1 & -1 \\ 2 & 0 \end{pmatrix}$, show that

$A^2 + 2I = A$; where I is the identity matrix of order 2.

(b)

Let $A = \begin{pmatrix} 2 & -3 & -5 \\ -1 & 4 & 5 \\ 1 & -3 & -4 \end{pmatrix}$, show that $A^2 = A$

Hence deduce that $(I - A)^2 = (I - A)$, where I is the identity matrix of order 3.

(c)

Let $A = \begin{pmatrix} \frac{-1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{-1}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{2}{3} & \frac{-1}{3} \end{pmatrix}$

Show that $AA^T = I$, where I is the identity matrix of order 3.

Q3 Consider 3×3 matrix A,

$$A = \begin{pmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{pmatrix}$$

(a) Find AA^T

(b) Find the inverse of the matrix A using Gaussian elimination method.

Q4

(a) Given that $\tan\theta = \frac{3}{4}$, θ in quadrant I, and $\tan\alpha = \frac{12}{5}$, α in quadrant I. Find

i) $\sin(\theta+\alpha)$ ii) $\cos(\theta-\alpha)$ iii) $\tan(\theta-\alpha)$

Give exact answers and show all your work.

(b) Sketch the graph of $y = -\sin^2 x$ in the period $0 \le x \le 2\pi$.

(c) Answer the following problems

i) The angle of elevation (upward angle from a horizontal level) of the top of a tower looking from a point 120m away from the tower's base is 60°. Find the height of the tower.

ii) Find the height of a chimney if the angle of elevation of its top changes from 30° to 45° as the observer advances 70m toward its base.

Q5

(a) Let $a = \sec\theta - \tan\theta$, where θ is not an odd multiple of $\frac{\pi}{2}$ and $a \ne 0$.

Show that, $\sec\theta + \tan\theta = \frac{1}{a}$

Deduce that, $\cos\theta = \frac{2a}{1+a^2}$ and $\sin\theta = \frac{1-a^2}{1+a^2}$

(b) Prove the following.

i) $\dfrac{\cos(A+B)+\cos(A-B)}{\sin(A+B)+\sin(A-B)} = \cot A$

ii) $\dfrac{2\sin^2 x}{2\tan x - \sin 2x} = \cot x$

iii) $\dfrac{\sin 75^\circ - \sin 15^\circ}{\cos 75^\circ + \cos 15^\circ} = \dfrac{1}{\sqrt{3}}$

(c) Find the general solution of the following equation $2\sin^2\theta - 3\sin\theta + 1 = 0$ in the range $0^0 \le \theta \le 360^0$.

Q6

(a) Find the following limits

i) $\lim_{x\to 0} \dfrac{\tan x + x}{\sin x}$

ii) $\lim_{x\to 5} \dfrac{x^2 - 25}{\sqrt{x-1} - 2}$

iii) $\lim_{x\to 3} \dfrac{x^2 - 5x + 6}{x^2 - 2x - 3}$

(b) i) Obtain a root of the equation $x^3 + x^2 - 3x - 3 = 0$, lying between 1 and 2 using method of false position(Regula Falsi), three times in succession.

ii) By using Newton-Raphson method, find the root of $x^3 - 1.5x^2 + 0.005 = 0$, which is near to $x = 0.1$ with three iterations.

Q7

(a) Find first derivatives of the following from first principles. Show all steps.

i) $x^2 + 3x + 2$ ii) $\sin x$

(b) Find $\dfrac{dy}{dx}$ of,

i) $y = \dfrac{1}{2}\left(\sqrt{1+x^2} - x\right)^2$ ii) $y = x^2 \cos^2 x$

(c) If $y = \sqrt{1+\sin x}$ show that

$$2\sqrt{1+\sin x}\,\frac{dy}{dx} = \cos x$$

Q8

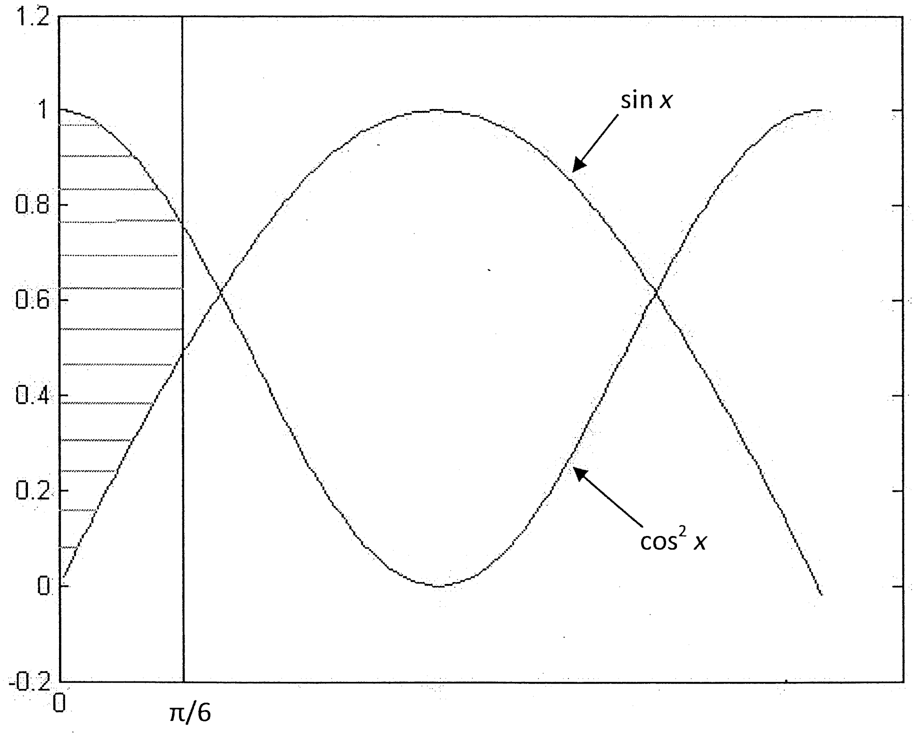
(a) Evaluate the following.

i) $\int (\sin x - \cos x)dx$ ii) $\int \cos(3x-2)dx$

(b) Find the exact value of the following.

i) $\int_0^2 \frac{1}{x^2+4}dx$ ii) $\int_0^4 \frac{1}{\sqrt{16-x^2}}dx$

(c) Consider the following figure with two curves.



i) Write an equation to find the shaded area of the figure by <u>integration</u> method.
ii) Hence, find the shaded area of the figure.