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The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination – 2009/2010
Level 04-Applied Mathematics
AMU 2185/AME 4185 – Numerical Methods I

Duration :- Two and Half Hours

Date :- 05-07-2010. **Time:- 9.30 a.m. – 12.00 noon**

Answer Four Questions Only

(1) (a) Explain briefly the following.

(i) Limiting absolute error.

(ii) Truncation error.

(b) A simple pendulum experiment is conducted to measure the gravitational acceleration (g). The formula used is $\frac{t}{N} = 2\pi\sqrt{\frac{l}{g}}$, where l - the length of the string, t- time taken and N- number of oscillations. The experiment measurements l = 60 cm, t =124.3 seconds and N = 80 were recorded with possible errors of $\pm$ 0.01 in each of the measurements of t and l. Find g and the limiting error.

(c) (i) Find the Taylor expansion polynomial of degree n about the origin for the function

$$f(x) = (1-x)^{\frac{1}{2}} \quad , \qquad -1 < x < 1 .$$

(ii) Write the above polynomial with the error of 4^{th} degree.

(iii) Calculate the truncation error (maximum error of 4^{th} degree) in part (ii) when $x = 0.1$.

(2) (a) Show by the graphical method that the equation $\alpha = 1 - \sin\alpha$ has a unique solution in the range of $\left(0, \frac{\pi}{2}\right)$. Tabulate the value of $\sin\alpha$, $1-\alpha$ and obtain the numerical solution of the above equation correct to fourth decimal place.

(b) Estimate the number of iterations that will be required to find the real root of $x^3+x^2+3x-3=0$ to 6 decimal places that converges according to the method of,

(i) Bisection

(ii) Simple iterative

(3) (a) What is the geometrical interpretation of the Newton's formula for solving a non-linear equation $f(x)=0$.

(b) With the usual notation, prove that the condition for convergences of the Newton's method is $\left|f(x^*)f''(x^*)\right|<\left|f'(x^*)\right|^2$ where x^* is a solution.

(c) Show that the equation $x^2+5x-1=0$ has a real root in (0.1. 0.2). Derive a simple iterative scheme that can be expected to converge to this point. To what decimal place will the result of iterations be accurate, if iterations are stopped at convergences to four decimal places.

(4) (a) Derive Newton's backward difference formula.

(b) The table gives the temperature in the shade at a particular location.

Time	1000h	1100h	1200h	1300h	1400h	1500h
Temperature	16	19	21	20	18	13

Use Newton's backward formula to find the temperature of steam at the time 1430h.

(c) Prove the following.

(i) $1+\Delta=(E-1)\nabla^{-1}$

(ii) $\frac{\nabla}{\Delta}=(1+\Delta)^{-1}$

(iii) $\Delta^2=E^2-2E+1$

(iv) $E^{\frac{1}{2}}\Delta=E^{-\frac{1}{2}}\nabla$

(v) $\Delta=E\nabla$

(5) (a) Derive Horner's scheme for division of monic n^{th} degree polynomial

$f(x) = x^n + a_1x^{n-1} + a_2x^{n-2} + a_3x^{n-3} + ... + a_{n-1}x + a_n$ by a factor $(x - \alpha)$.

(b) The oil level (h) in a container is given by $f(h) = h^3 + 2h^2 + 10h - 20$. .

(i) Find all roots of *f(h)* correct to 3 decimal places.

(ii) What is the maximum oil level in the tank?

(6) (a) Write down the n^{th} order Lagrange's interpolation polynomial *P(x)* for the data set

$(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$.

(b) With the usual notation, prove that the error of interpolation by Lagrange's method

is $\frac{\pi(x)}{(n+1)!} f^{n+1}(c)$, where $c \in (x_0,\ x_n)$ and $\pi(x) = \prod_{i=0}^{n} (x - x_i)$.

(c) Use Lagrange's method to find $f(0.18)$ based on the following table.

x	0.05	0.10	0.15	0.2
f(x)	0.05637	0. 11246	0.16800	0.2270