00141

The Open University of Sri Lanka
B.Sc. / B.Ed. Degree Programme – Level 04
Final Examination – 2009/2010
Applied Mathematics
AMU 2184/AME 4184 – Newtonian Mechanics

Duration :- Two and Half Hours

Date :- 01-07-2010 **Time :-1.30 p.m. – 4.00 p.m.**

Answer Four Questions Only.

1. A particle of mass m is projected vertically upwards with speed u_0 under gravity in a medium which exerts a resisting force of magnitude mkv, where v is the speed of the particle at time t and k is a constant. Obtain an expression for the greatest height h attained by the particle in terms of u_0, k, g and deduce that

   $h = (u^2/g)\left[\lambda - \ln(1+\lambda)\right]$, where $u = u_0/\lambda$ is the terminal velocity of the particle.

   Find also the speed with which the particle will return to the point of projection.

2. A particle P of mass m is attached to the mid-point of an unstretched elastic string of natural length a and modulus mg. The string, with the mass attached, is then stretched between two points in the same vertical line, distant $2a$ apart. Find the position of equilibrium of P.

   If the particle P is slightly displaced from its equilibrium position in the vertical direction, show that the ensuing motion is simple harmonic with period $\pi\sqrt{a/g}$.

   Also, show that, if the particle P is slightly displaced from its equilibrium position in a horizontal direction, the ensuing motion is approximately simple harmonic with period $\pi\sqrt{15a/7g}$.

3. (a) Establish the formula $\underline{F}(t) = m(t)\dfrac{d\underline{v}}{dt} + \dfrac{dm}{dt}\underline{u}$ for the motion of a particle of varying mass $m(t)$ moving with velocity $\underline{v}$ under a force $\underline{F}(t)$, matter being emitted at a rate $\dfrac{dm}{dt}$ with velocity $\underline{u}$ relative to the particle.

   (b) At time t, the mass of a rocket is $M(1-kt)$, where M and k are constants. At this instant the rocket is moving with speed v vertically upwards near the Earth's surface against constant gravity. Burnt fuel is expelled vertically downwards at speed u relative to the rocket.

(i) Show that $(1-kt)\frac{dv}{dt}=ku-g(1-kt)$.

(ii) Given that $v=0$ when $t=0$, find v in terms of g, u, k and t.

(iii) Derive an expression for the distance traveled in terms of g, u, k and t.

4. (a) With the usual notation show that the velocity and acceleration components in plane polar coordinates are given by $\underline{v}=\dot{r}\underline{e}_r+r\dot{\theta}\underline{e}_\theta$ and $\underline{a}=(\ddot{r}-r\dot{\theta}^2)\underline{e}_r+\frac{1}{r}\frac{d(r^2\dot{\theta})}{dt}\underline{e}_\theta$.

(b) A particle of unit mass moves in a straight line l with acceleration directed towards a fixed point O on l having magnitude $\left(\frac{\mu}{x^2}-\frac{\lambda}{x^3}\right)$, x being the distance from the point O. The particle starts from rest at a point A distant a from O. Show that it oscillates between point A and a point distant $\frac{\lambda a}{2a\mu-\lambda}$ from O and that the periodic time is given by $\frac{2\pi\mu a^3}{(2a\mu-\lambda)^{\frac{3}{2}}}$.

5. (a) With the usual notation show that the equation of the orbit of a particle moving under a central force F per unit mass is given by $\frac{F}{h^2u^2}=u+\frac{d^2u}{d\theta^2}$.

(b) A particle, of mass m, is projected from a point A, at a distance a from a fixed point O, with a velocity $\sqrt{\mu}/a$, in the direction AP where the angle OAP is 45°. It is subject to a force $\mu m/r^3$ directed towards O, where r is the distance from O. Show that the orbit of the particle has the polar equation $r=ae^{-\theta}$

6. (a) A particle P moves with speed V acceleration μ/r^2 directed towards a fixed point S, where $r=SP$. Prove that its orbit is an ellipse, a hyperbola or a parabola according as $V^2 \lesseqgtr 2\mu/r$, where V is the speed of the particle.

(b) A Particle of mass m moving in a circle of radius c under an attractive force μ/r^2 per unit mass towards the centre, collides and coalesces with a particle of mass λm which is at rest. Show that the orbit of the combined mass is an ellipse with major axis $c\operatorname{cosec}^2\alpha$, latus rectum $4c\operatorname{cosec}^2\alpha$, and eccentricity $-\cos 2\alpha$, where $\sec^2\alpha=2(1+\lambda)^2$.