The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme – Level 05
Final Examination – 2009/2010
Applied Mathematics
AMU 3184/AME 5184 – Dynamics

Duration :- Two and a Half Hours

Date :- 19-01-2010. **Time :- 9.30 a.m. – 12.00 noon**

Answer Four Questions Only.

1. (a) Obtain the velocity and acceleration components in intrinsic coordinates, for a particle moving along a curve in a plane.

(b) A smooth wire in the form of $s = 4a\sin\psi$, $\left(-\frac{\pi}{2} < \psi < \frac{\pi}{2}\right)$ is fixed in a vertical plane, the vertex O being the lowest point of the wire. A bead of mass m, which can slide freely on the wire is released from rest at the point where $\psi = \pi/3$. Find the period of oscillation of the bead, and show that the reaction of the wire at a point where the tangent makes an angle ψ with the horizontal is $mg\sec\psi\left(8\cos^2\psi - 1\right)/4$.

2. (a) Show that, in cylindrical polar coordinates the velocity and acceleration vectors are given by $\dot{\underline{r}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + \dot{z}\underline{k}$ and $\ddot{\underline{r}} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{r} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\theta} + \ddot{z}\underline{k}$ respectively.

(b) A particle of mass m moves on the smooth inner surface of the paraboloid of revolution $r^2 = 2az$, whose axis is vertical and vertex downwards. Find the angular momentum of the particle about OZ if the particle describes a horizontal circle of radius a with constant speed. While the particle is describing the above circle, it receives an impulse $m\sqrt{2ag}$ along the surface of the paraboloid in a vertical plane through the axis. Show that in the subsequent motion the path of the particle lies between two horizontal planes.

3. (a) In the usual notation, derive Lagrange's equations $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} = Q_j$, $j = 1, 2, \ldots, n$; explain the meaning of the symbols T and Q.

(b) Write down the form taken by the above equations, for a system with conservative forces.

(c) AB is a straight smooth wire fixed at a point A on a vertical axis OA such that AB rotates about OA with constant angular velocity ω. A bead of mass m moves on the wire. Given that $OA = h$.

(i) Set up the Lagrangian for the system.

(ii) Obtain the Lagrange's equations of motion.

(3) (a) Let $f_1(x), f_2(x), f_3(x),\ldots$ be a set of real-valued functions , which are orthogonal with respect to the weight function $p(x)$ on the interval $a \le x \le b$. If $h_m(x) = \sqrt{p(x)}\, f_m(x)$; $(m = 1, 2, 3,\ldots)$, then show that $h_1(x)$, $h_2(x), h_3(x)\ldots$are orthogonal on the interval $a \le x \le b$.

(b) (i) Let $f_1(x) = a$, $f_2(x) = 1 - bx$ and $f_3(x) = 1 - cx + dx^2$ where a, b, c and d are non-zero constants. Suppose that $f_1(x), f_2(x), f_3(x)$ are orthogonal in the interval $0 < x < \infty$ with respect to the weight function $p(x) = e^{-x}$. If $\|f_1(x)\| = 1$, find the values of a, b, c and d.

(ii) Show that $\{f_1(x), f_2(x), f_3(x)\}$ forms an orthonormal set in the interval $0 < x < \infty$ with respect to the weight function $p(x) = e^{-x}$.

(4) Consider the boundary value problem

$$\frac{d^2y}{dx^2} + \mu y = 0$$

$$y(-2) = y(2)$$

$$y'(-2) = y'(2)$$

(i) Show that this is a Sturm-Liouville problem.

(ii) Find the eigenvalues and eigenfunctions of the problem.

(iii) Obtain a set of functions which are orthonormal in the interval $-2 \le x \le 2$.

(5) Let $J_n(x)$ be the Bessel function of order n given by the expansion

$$e^{\frac{x}{2}\left(t-\frac{1}{t}\right)} = \sum_{n=-\infty}^{\infty} J_n(x) t^n .$$

Verify each of the following identities for n=1, 2, 3,….

(i) $J_{n+1}(x) = \frac{2n}{x} J_n(x) - J_{n-1}(x)$

(ii) $J'_n(x) = \frac{1}{2}[J_{n-1}(x) - J_{n+1}(x)]$

(iii) $J'_n(x) + \frac{n}{x} J_n(x) = J_{n-1}(x)$

(iv) $\frac{d}{dx}[J_n^2(x)] = \frac{x}{2n}[J_{n-1}^2(x) - J_{n+1}^2(x)]$

(6) Solve the following boundary value problem with mixed boundary conditions.

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0, \quad 0<x<a, \quad 0<y<b.$$

$$\frac{\partial u}{\partial x}(0, y)=0, \quad 0<y<b.$$

$$\frac{\partial u}{\partial x}(a, y)=0, \quad 0<y<b.$$

$$u(x, 0)=0, \quad 0<x<a.$$

$$u(x, b)=1, \quad 0<x<a.$$