The Open University of Sri Lanka
B.Sc /B.Ed. Degree Programme/ Continuing Education Programme
Final Examination-2009/2010
AMU 3182/AME 5182-Mathematical Methods I
Level 05-Applied Mathematics

Duration: Two and half hours

Date:06-01-2010 **Tme:9.30 a.m-12.00 noon**

Answer four questions only.

01. (a) Let A be a matrix of order $n\times n$. Suppose that A has a linearly independent eigen vectors $a_1, a_2, \ldots, a_n$ corresponding to the eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$ respectively. Show that the general solution of the system of differential equations $\dot{X} = AX$ is $X = \sum_{r=1}^{n} C_r a_r e^{\lambda_r t}$, where C_r are arbitrary constants.

(b) Find the general solution of the system of simultaneous differential equations given below.

(i) $\ddot{x}_1 = -5x_1 - 12x_2 + 6x_3$

$\ddot{x}_2 = x_1 + 5x_2 - x_3$

$\ddot{x}_3 = -7x_1 - 10x_2 + 8x_3$

(ii) $\dot{x}_1 = 5x_1 - 4x_2$

$\dot{x}_2 = 2x_1 + x_2$

02. (a) Which of the following could be solved by separation of variables? Give your reasons for your answer.

(i) $\dfrac{\partial^2 u}{\partial x \partial y} + 2u = 0$

(ii) $x^2 \dfrac{\partial^2 u}{\partial x^2} + y \dfrac{\partial^2 u}{\partial y^2} = 0$

(iii) $\dfrac{\partial^2 u}{\partial x^2} + 2uxy + \dfrac{\partial^2 u}{\partial y^2} = 0$

(b) Solve the following heat problem that satisfy the given initial conditions.

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2},$$

$u(x,0) = f(x)$, $u(0,t) = 0$ and $u(L,t) = 0$, where

$$f(x) = 12\sin\left(\frac{9\pi x}{L}\right) - 7\sin\left(\frac{4\pi x}{L}\right).$$

03. (a) Solve the following differential equations:

(i) $6x^2\dfrac{d^2y}{dx^2}-5x\dfrac{dy}{dx}-10y=0$.

Also, find the particular solution for which y=5 and $\dfrac{dy}{dx}=3$, when x=1.

(ii) $x^2\dfrac{d^2y}{dx^2}-x\dfrac{dy}{dx}+y=4\sin(\ln x)$, x>0.

(b) Find a particular sinusoidal solution of the following system of differential equations:

$$4\ddot{x}_1=8x_1-5x_2+\sin 2t$$
$$4\ddot{x}_2=10x_1-7x_2+2\cos 2t$$

04. (a) For each of the following equations, classify the equation and find its general solution:

(i) $2\dfrac{\partial^2u}{\partial x^2}-\dfrac{\partial^2u}{\partial x\partial y}-6\dfrac{\partial^2u}{\partial y^2}=0$.

(ii) $\dfrac{\partial^2u}{\partial x^2}-2\dfrac{\partial^2u}{\partial x\partial y}+\dfrac{\partial^2u}{\partial y^2}=0$.

(b) Consider the partial differential equation

$$y^2\frac{\partial^2u}{\partial x^2}-x^2\frac{\partial^2u}{\partial y^2}-y^2\left(4x+\frac{1}{x}\right)\frac{\partial u}{\partial x}+x^2\left(4y+\frac{1}{y}\right)\frac{\partial u}{\partial y}=0\ ,\ (y\neq 0),(x\neq 0).$$

(i) Show that the characteristics are defined by the pair of ordinary differential equations

$$\frac{dy}{dx}=\pm\frac{x}{y}$$

and hence find the characteristics and sketch them in the *oxy* plane.

(ii) Show that the equation in (i) can be rewritten in standard form as

$$\frac{\partial^2u}{\partial\eta\partial\phi}-\frac{\partial u}{\partial\eta}=0,$$

where ϕ and η are the characteristic variables, and thus find the general solution of the equation.

05. (a) Using integrating factor method, find the general solution of each of following differential equations:

(i) $\frac{\partial u}{\partial y}+\frac{1}{y}u=e^{x}+e^{y}$

(ii) $\frac{\partial u}{\partial y}+\frac{2}{y}u=2(1+\ln y)$

(b) Find the general solution of the pair of simultaneous partial differential equations:

$$\frac{\partial u}{\partial x}=\frac{2x}{x^2-y^2}+4x(x-y)+2(x-y)^2$$

$$\frac{\partial u}{\partial y}=\frac{-2y}{x^2-y^2}-4x^2+4xy+3y^2.$$

(c) Find the equations of the characteristic curves for the partial differential equation

$$\frac{\partial u}{\partial x}-x\frac{\partial u}{\partial y}+u=2x+y \qquad (x>0, y>0)$$

Hence or otherwise, find the general solution of the given partial differential equation.

06. (a) Find the general solution of the system of simultaneous differential equations:

$$\frac{dx_1}{dt}=2x_1+3x_2+2e^{3t}$$

$$\frac{dx_2}{dt}=5x_1+4x_2-3.$$

(b) Use Euler's method to obtain recurrence relations in a form suitable for generating approximations for the system of equations given in part (a), where $x_1=2$ and $x_2=1$ when $t=0$.

(c) Use the recurrence relations obtained in part (b) with a step-length of 0.1, to calculate approximations to $x_1(0.2)$ and $x_2(0.2)$.