

The Open University of Sri Lanka  
 B.Sc/B.Ed. Degree, Continuing Education Programme  
 Final Examination 2011/2012  
 Level 04 Applied Mathematics  
 APU2140/APE4140 – Statistical Distribution Theory



Duration: - Two Hours.

DATE: - 31-12-2011.

Time: - 9.30 a.m. – 11.30 a.m.

Non programmable calculators are permitted. Statistical tables are provided

Answer FOUR questions only.

- (1) A company that produces a certain electrical product claims that the life time  $X$  (years) has the density function

$$f(x) = \begin{cases} x^2 & ; 0 \leq x < 1 \\ \frac{7-3x}{4} & ; 1 \leq x < \frac{7}{3} \\ 0 & ; \text{otherwise} \end{cases}$$

- (i) Find the expected life time of a randomly selected electrical product.
- (ii) Find the standard deviation of the lifetime of a randomly selected product.
- (iii) Find the cumulative distribution function of  $X$ .
- (iv) Find the probability that a randomly selected product will not fail within two years.
- (v) Find the highest lifetime of the lowest 50% of lifetimes of the product.
- (2) A particular text book is published by two publishers namely A and B. The text book has 100 pages and typographical errors could occur on these pages. Suppose that the text book published by A has exactly 3 such errors randomly located on those pages and the text book published by B has exactly 8 such errors randomly located on those pages. Assume that number of errors that could occur on a page is not more than one.
- (a) Mr. Perera has bought a book which was published by A. Suppose that Mr. Perera has read some distinct pages randomly in the book.
- (i) Find the probability that a random selection of 10 distinct pages in the book will contain no errors.
- (ii) Find the probability that 10 randomly selected distinct pages will contain at least two errors.

- (b) Mr. Dias has borrowed a text book and has read ten distinct random pages. He has found 2 errors out of 10 distinct pages. What is the probability that the book read by Mr. Dias was published by the publisher B. Assume that the both publishers A and B have published equal no of books.
- (3) The ABC Company has two showrooms in Colombo city limits. One is located at Petta and the other one located at Maradana. Both of these showrooms sell Sun brand electric fans. Let  $X$  be the number of Sun brand electric fans sold per day at the Petta showroom and let  $Y$  be the number of Sun brand electric fans sold per day at the Maradana showroom. The following table shows the joint probabilities, according to the past data.

$P(x,y)$		$x$		
		0	1	2
$y$	0	0.03	0.15	0.12
	1	0.05	0.15	0.16
	2	0.02	0.2	0.12

- (i) Find the marginal distribution functions of  $X$  and  $Y$ .
- (ii) The sales manager of the ABC Company claims that the sales of Sun brand electric fans at Petta and Maradana showrooms are independent. Do you agree with the sales manager's claim? Justify your answer.
- (iii) Find the total expected sales of Sun brand fans per day at two showrooms.
- (iv) Assume that these two showrooms open at 9.00a.m. and close at 4.00 p.m. on week days. On a particular weekday salesman of the Maradana showroom has sold their first Sun brand electric fan at 1.00 p.m. What is the probability of no sales of Sun brand fans at Petta showroom on that day.

(4)

(a) Random Variable  $X$  has range  $\{0,1,2,3,\dots\}$  and the mass function

$$P_X(x) = \frac{e^{-\lambda} \lambda^x}{x!} \text{ for all } x.$$

Let  $M_x(t)$  is the moment generating function of  $X$ .

(i) Show that  $M_x(t) = e^{\lambda(e^t-1)}$

(ii) Using part (i) show that  $E(X) = \lambda$  and  $\text{Var}(X) = \lambda$

(b) Suppose the number of babies born during an 8-hour shift at a hospital's maternity wing follows the distribution given in part (a) with a mean of 24.

(i) Find the probability that two babies are born during a particular 1-hour period in this maternity wing.

(ii) Give another distribution which can be used to approximate the number of babies born during an 8-hour shift. Clearly state the distribution and the parameters that you suggest. Hence find the probability that minimum of 34 babies are born during a particular 8-hour session in this maternity wing.

(5)

(a) Suppose  $X_1 \sim N(2,9)$   $X_2 \sim N(2,4)$   $X_3 \sim \text{exp}(6)$   $X_4 \sim \text{exp}(6)$   $X_5 \sim \text{gamma}(3,6)$ 

Assume all these variables are independent. Calculate the following probabilities.

(i)  $Pr(X_1 + 3X_2 \geq 20)$

(ii)  $Pr(X_3 + X_4 + X_5 \leq 1)$

- (b) A machine is set to produce floor tile with mean diameter 35cm. Each day a random sample of 100 floor tiles are selected and the diameters are accurately measured. If the sample mean diameter lies outside the range 34.98 cm to 35.02 cm then it will be taken as evidence that mean diameter of the floor tiles produced is not 35 cm. The machine will then be stopped and adjustments are made. Assuming that the diameter has a standard deviation of 1 cm, find the probability of the machine being stopped unnecessarily for adjustments on a randomly chosen day.
- (6) The specifications on an electronic component in a target-acquisition system are that its lifetime must exceed 5000 hours. When the production process is in control the lifetime is normally distributed with mean 7500h and standard deviation 1000h . The manufacturer earns Rs. 1000 a unit product however defective unit must be replaced at a cost of RS 750 to the manufacturer.
- Find the fraction of products within the specification when the process is in control.
  - How many unit products are within the specifications out of 10000 unit products when the process is in control?
  - Find the expected total earning of the manufacturer out of 10000 unit products when the process is in control.
  - Find the mean lifetime of a randomly selected unit product, if the total number of unit products which were within specification is 7881 out of 10000 unit products. Assume that the standard deviation of the lifetime of the unit product is 1000h.

**Left tail values of Standard Gamma Table**

$$W - \text{gamma}(\alpha, 1)$$

This table contain the probabilities  $\Pr(W \leq w)$

w	$\alpha$					
	1	2	3	4	5	6
1	0.393469	0.264241	0.080301	0.018988	0.00366	0.000594
2	0.632121	0.593994	0.323324	0.142877	0.052653	0.016564
3	0.77687	0.800852	0.57681	0.352768	0.184737	0.083918
4	0.864665	0.908422	0.761897	0.56653	0.371163	0.21487
5	0.917915	0.959572	0.875348	0.734974	0.559507	0.384039
6	0.950213	0.982649	0.938031	0.848796	0.714943	0.55432