The Open University of Sri Lanka
B.Sc/B.Ed. Degree, Continuing Education Programme
Final Examination 2011/2012
Level 04 Applied Mathematics
AMU2182/AME4182– Statistics I

Duration: - Two Hours.

DATE: - 31-12-2011. **Time: - 9.30 a.m. – 11.30 a.m.**

Non programmable calculators are permitted. Statistical tables are provided

Answer FOUR questions only.

(1) A company that produces a certain electrical product claims that the life time X (years) has the density function

$$f(x) = \begin{cases} x^2 & ; \quad 0 \leq x < 1 \\ \frac{7-3x}{4} & ; \quad 1 \leq x < \frac{7}{3} \\ 0 & ; otherwise \end{cases}$$

(i) Find the expected life time of a randomly selected electrical product.

(ii) Find the standard deviation of the lifetime of a randomly selected product.

(iii) Find the cumulative distribution function of X.

(iv) Find the probability that a randomly selected product will not fail within two years.

(v) Find the highest lifetime of the lowest 50% of lifetimes of the product.

(2) (a) According to the past data, the probability of not raining to a stadium in a certain area on a given day during April is 0.95. A netball tournament is going to be held in this stadium from 15th April 2012 to 18th April 2012.

(i) What is the probability that there will be no rain during the tournament?

(ii) What is the probability that there will be rain on two days during the tournament?

(iii) Find the probability that there will be rain on the second and the third days of the tournament.

(b) A particular company has 3 counters namely A,B and C. On average A serves 45%, B serves 30% and C serves 25% of the customers. The probability that A makes a mistake is 0.14 and the respective probabilities for B and C are 0.2 and 0.1. One day the manger receives a complaint regarding an error in a customer's account. What is the probability that the error was committed by the counter A?

(3) The ABC Company has two showrooms in Colombo city limits. One is located at Petta and the other one located at Maradana. Both of these showrooms sell Sun brand electric fans. Let X be the number of Sun brand electric fans sold per day at the Petta showroom and let Y be the number of Sun brand electric fans sold per day at the Maradana showroom. The following table shows the joint probabilities, according to the past data.

$P(x,y)$		x		
		0	1	2
y	0	0.03	0.15	0.12
	1	0.05	0.15	0.16
	2	0.02	0.2	0.12

(i) Find the marginal distribution functions of X and Y.

(ii) The sales manager of the ABC Company claims that the sales of Sun brand electric fans at Petta and Maradana showrooms are independent. Do you agree with the sales manager's claim? Justify your answer.

(iii) Find the total expected sales of Sun brand fans per day at two showrooms.

(iv) Assume that these two showrooms open at 9.00a.m. and close at 4.00 p.m. on week days. On a particular weekday salesman of the Maradana showroom has sold their first Sun brand electric fan at 1.00 p.m. What is the probability of no sales of Sun brand fans at Petta showroom on that day.

(4)

(a) Random Variable X has range {0,1,2,3,.....} and the mass function

$$P_X(x) = \frac{e^{-\lambda}\lambda^x}{x!} \text{ for all } x.$$

Let $M_x(t)$ is the moment generating function of X.

(i) Show that $M_x(t) = e^{\lambda(e^t-1)}$

(ii) Using part (i) show that $E(X) = \lambda$ and $Var(X) = \lambda$

(b) Suppose the number of babies born during an 8-hour shift at a hospital's maternity wing follows the distribution given in part (a) with a mean of 24.

(i) Find the probability that two babies are born during a particular 1-hour period in this maternity wing.

(ii) Give another distribution which can be used to approximate the number of babies born during an 8-hour shift. Clearly sate the distribution and the parameters that you suggest. Hence find the probability that minimum of 34 babies are born during a particular 8- hour session in this maternity wing.

(5)

(a) In the case of snake bites, often, lethal doses are not imparted. Suppose 80% of the snake bite cases recover even without treatment. What is the probability that out of 100 cases untreated more than 85 will survive.

(b) Suppose the life in hours of a light bulb of a make is normally distributed with mean 100 hours. If the manufacturer wants to ensure that 90% of the bulbs have life exceeding 80 hours, find the variance that the distribution must have.

(c) A machine is set to produce floor tile with mean diameter 35cm. Each day a random sample of 100 floor tiles are selected and the diameters are accurately measured. If the sample mean diameter lies outside the range 34.98 cm to 35.02 cm then it will be taken as evidence that mean diameter of the floor tiles produced is not 35 cm. The machine will then be stopped and adjustments are made. Assuming that the diameter has a standard deviation of 1 cm, find the probability of the machine being stopped unnecessarily for adjustments on a randomly chosen day.

(6) The specifications on an electronic component in a target-acquisition system are that its lifetime must exceed 5000 hours. When the production process is in control the lifetime is normally distributed with mean 7500h and standard deviation 1000h . The manufacturer earns Rs. 1000 a unit product however defective unit must be replaced at a cost of RS 750 to the manufacturer.

(i) Find the fraction of products within the specification when the process is in control.

(ii) How many unit products are within the specifications out of 10000 unit products when the process is in control?

(iii) Find the expected total earning of the manufacturer out of 10000 unit products when the process is in control.

(iv) Find the mean lifetime of a randomly selected unit product, if the total number of unit products which were within specification is 7881 out of 10000 unit products. Assume that the standard deviation of the lifetime of the unit product is 1000h.