The Open University of Sri Lanka
B.Sc/B.Ed. Degree Programme
Final Examination - 2011/2012
Applied Mathematics-Level 05
APU3240 – Numerical Methods

Duration:- Three hours.

Date:-23.12.2011 **Time: - 1.30p.m.-4.30p.m.**

Answer **Five** Questions only.

1. (a) Find the maximum relative error of $u=\dfrac{3x^2y^2}{z}$ at $x=y=z=1$, when the error in x, y and z are $\delta x=\delta y=\delta z=0.01$ respectively.

(b) (i) Construct an iterative scheme that satisfies the condition for convergence to solve the equation $1+\ln x-\dfrac{x}{2}=0$.

(ii) Estimate the number of iterations that will be required to find a solution for the above equation, correct to 2 decimal places, by means of your iterative scheme in (i). Hence find the root of the equation.

2. (a) Given a set of data in the form (x_i, y_i, y_i'), $i=0,1,\ldots,n$. Write the expression for the Hermite interpolation formula.

Find the Hermite interpolation polynomial for the following data:

x	$y(x)$	$y'(x)$
1	3	0
2	0	1
3	-1	0

(b) Find the cubic spline corresponding to the interval $(2, 3)$ from the following table

x	1	2	3	4
y	1	2	5	11

Hence compute $y(2.5)$.

3. (a) Let $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$ be the given $(n+1)$ points. With the usual notations, derive the Newton's general interpolation formula using divided differences.

(b) Compute the divided difference table for the following set of data generated from the function $f(x) = \ln(x+3)$.

x	-1	-0.5	0	0.4	0.8
$\ln(x+3)$	0.6931	0.9163	1.0986	1.2238	1.3350

Using the table, find $f(x)$ as polynomial in x. Hence, approximate the value of $\ln 3.5$. Also, find the value of $\frac{1}{3}$.

4. (a) Write the Trapezoidal rule for the integral $\int_{x_0}^{x_n} f(x)\,dx$, when the interval $[x_0, x_n]$ is divided into n equal subintervals.

Hence, find the approximate value of $\int_{1.2}^{1.6} e^{-x^2}\,dx$, with subinterval size $h = 0.1$.

(b) Given that the interval (a, b) is divided into n equal subintervals with each of length h and the interval (c, d) is divided into m equal subintervals with each of length k, write an expression to evaluate $\int_c^d \int_a^b f(x, y)\,dx\,dy$.

Hence, evaluate the approximate value of $I = \int_0^1 \int_0^1 xe^y\,dx\,dy$ with $h = k = 0.5$.

5. (a) Write down the third order Taylor series method for an initial value problem.

(b) Solve the system of differential equations $\frac{dy}{dx} = x + z$ and $\frac{dz}{dx} = x - y^2$ at $x = 0.1$ and $x = 0.2$, by using the third order Taylor series method with $h = 0.1$. Given that $y(0) = 2,\ z(0) = 1$.

6. (a) Derive Euler's method to find an approximate solution to the differential equation $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(b) Consider the initial value problem

$$\frac{dx}{dt} = \frac{x^2 + t^2}{2x}, \quad x(1) = 1.$$

Using the (i) Euler's method (ii) Modified Euler's method with step size $h = 0.2$, find the approximate value of $x(1.6)$. Give your answer correct to three decimal places.

7. (a) Write down the fourth order Runge – Kutta method to find an approximate solution to the differential equation $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(b) Use the fourth order Runge–Kutta method to solve the differential equation $\frac{dy}{dx} = \frac{y^2 - 2x}{y^2 + x}$, subject to the initial condition $y(0) = 1$ and assuming $h = 0.1$. Hence find the value of y at $x = 0.1,\ 0.2,\ 0.3$. Give your results to four decimal places.

8. (a) (i) Derive Picard's method to find an approximate solution to the differential equation $\frac{dy}{dx} = f(x, y)$ subject to the initial condition $y(x_0) = y_0$.

(ii) Consider the initial value problem

$$\frac{dy}{dx} = 2x - y^2 \text{ where } y = 0 \text{ at } x = 0.$$

Applying Picard's method, finds the first three successive approximations for the above problem.

(b) A Milne's Predictor- Corrector method applied to $\frac{dy}{dx} = f(x, y)$ is given by

$$\text{Predictor}: y_{n+1,p} = y_{n-3} + \frac{4h}{3}\left(2y'_{n-2} - y'_{n-1} + 2y'_n\right)$$

$$\text{Corrector}: y_{n+1,c} = y_{n-1} + \frac{h}{3}\left(y'_{n-1} + 4y'_n + y'_{n+1}\right).$$

where $y'_n = f(x_n, y_n)$.

Given $y' = \frac{1}{x+y}$ with $y(0) = 2, y(0.2) = 2.0933$, $y(0.4) = 2.1755$ and $y(0.6) = 2.2493$.

Find $y(0.8)$ using the above Milne's Predictor- Corrector method.