The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination-2010/2011
APU 2144- Applied Linear Algebra and Differential Equations
APPLIED MATHEMATICS-LEVEL 04

Duration: Two Hours.

Date:07.07.2011	**Time: 1.30 p.m.- 3.30p.m.**

Answer FOUR questions only.

(1) (i) Define the following:

(a) Minor of a matrix.

(b) Cofactor of a matrix.

(c) A non-singular matrix.

(ii) (a) Given two matrices B and C, what is the condition for the product BC to exist?

(b) Let $A=\begin{pmatrix} 2 & -2 & 10 \\ 3 & 5 & -6 \end{pmatrix}$, $B\begin{pmatrix} 1 & -3 & 0 & 4 \\ -2 & 5 & -8 & 9 \end{pmatrix}$ and $C=\begin{pmatrix} 8 & 5 & 3 \\ -3 & 10 & 2 \\ 2 & 0 & -4 \\ -1 & -7 & 5 \end{pmatrix}$.

Compute $3A+BC$.

(iii) Using the cofactor expansion compute the determinant of the matrix

$$\begin{pmatrix} 5 & -2 & 2 & 7 \\ 1 & 0 & 0 & 3 \\ -3 & 1 & 5 & 0 \\ 3 & -1 & -9 & 4 \end{pmatrix}.$$

(2) (i) Prove that a matrix and its transpose have the same eigen values.

(ii) Solve the following system of equations by calculating the inverse using elementary row operations.

$$
\begin{aligned}
x_1 + x_2 + x_3 + x_4 &= 0 \\
x_1 + x_2 + x_3 - x_4 &= 4 \\
x_1 + x_2 - x_3 + x_4 &= -4 \\
x_1 - x_2 + x_3 + x_4 &= 2
\end{aligned}
$$

(iii) Find the eigen values and corresponding eigen vectors of $A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$.

Determine whether the eigen vectors are orthogonal.

(3) (i) State the Cayley-Hamilton Theorem.

(ii) If $A = \begin{pmatrix} 1 & 5 \\ 3 & 4 \end{pmatrix}$, express $A^4 - 2A^2 - 4I$ as a linear polynomial in A.

(iii) Diagonalise the matrix $\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$ and hence find A^3.

(4) (i) Find the general solution of the system of simultaneous differential equations given below.

$$
\begin{aligned}
\dot{x}_1 &= x_1 + 3x_2 - 3x_3 \\
\dot{x}_2 &= -3x_1 + 7x_2 - 3x_3 \\
\dot{x}_3 &= -6x_1 + 6x_2 - 2x_3
\end{aligned}
$$

(ii) Find a solution of the system of equations given below.

$$
\begin{aligned}
\dot{x}_1 + 10x_1 + 6x_2 &= 6e^t \\
\dot{x}_2 + 6x_1 + 10x_2 &= 4e^t
\end{aligned}
$$

(5) (i) Find a sinusoidal solution of the following system of equations.

$$\ddot{x}_1 + 8\dot{x}_2 + x_1 = 8\sin 3t - 8\cos 3t$$
$$\ddot{x}_2 + \dot{x}_1 = -6\sin 3t - 3\cos 3t$$

(ii) Find the general solution of each of the differential equations

(a) $x^3\dfrac{d^3y}{dx^3} - 4x^2\dfrac{d^2y}{dx^2} + 8x\dfrac{dy}{dx} - 8y = 4\ln x$.

(b) $x^2\dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} + 2y = x^3$.

(6) Find all the eigenvalues and eigenfunctions for the following boundary value problem.

$$y'' + \lambda y = 0,$$
$$y(0) - y'(0) = 0,$$
$$y(\pi) - y'(\pi) = 0$$