The Open University of Sri Lanka
B.Sc./B.Ed Degree Programme – Level 04
Final Examination 2010/2011
Applied Mathematics
APU 2142 – Newtonian Mechanics I
Duration :- Two Hours

Date :-27.06.2011 **Time:-1.30 p.m. -3.30 p.m.**

Answer Four Questions Only.

1. A particle moving in a straight line, is subjected to a retardation of kv^n where v is the speed at time t and n is a positive constant. Find v as a function of t. Show that, if $n<1$, particle will come to rest at a distance $\frac{u^{2-n}}{k(2-n)}$ from the point of projection after a time $\frac{u^{1-n}}{k(1-n)}$ where u is the initial speed.

Discuss briefly what happens when

(a) $1<n<2$ (b) $n>2$.

2. With the usual notation, show that the velocity and acceleration components of a particle moving along a 2D curve in intrinsic coordinates, are given by $\underline{v}=\dot{s}\,\underline{t}$ and $\underline{a}=\ddot{s}\,\underline{t}+\frac{\dot{s}^2}{\rho}\,\underline{n}$.

A smooth wire in the form of an arch of a cycloid with intrinsic equation: $s=4a\sin\psi$, $-\frac{\pi}{2}\le\psi\le\frac{\pi}{2}$ is fixed in a vertical plane, the vertex O being the lowest point of the wire with the tangent at O is horizontal. A bead, of mass m, which can slide freely on the wire, is released from rest at the point where $\psi=\frac{\pi}{6}$. Write down the equations of motion for the bead.

(a) Find the periodic time of oscillation of the bead.

(b) Show that the normal contact force exerted by the wire on the bead at a point where the tangent makes an angle ψ with the horizontal is $\frac{1}{4}mg\sec\psi(8\cos^2\psi-3)$

3. With the usual notation show that the velocity and acceleration components in plane polar coordinates are given by $\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta$ and $\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + \frac{1}{r}\frac{d(r^2\dot{\theta})}{dt}\underline{e}_\theta$.

A particle, of mass m, is projected from a point A, at a distance a from a fixed point O, with a velocity $\frac{\sqrt{\mu}}{a}$, in the direction making an angle of 45° with OA. A force $\frac{\mu m}{r^3}$ directed towards O, where r is the distance from O acts on the particle. Show that the orbit of the particle has the polar equation $r = ae^{-\theta}$.

4. Establish the formula $\underline{F}(t) = m(t)\frac{d\underline{v}}{dt} - \frac{dm}{dt}\underline{u}$ for the motion of a particle of varying mass $m(t)$ moving with velocity $\underline{v}$ under a force $\underline{F}(t)$, matter being condensed at a rate $\frac{dm}{dt}$ with velocity $\underline{u}$ relative to the particle.

A particle P falls from rest under gravity in a straight line through a stationary cloud. The mass of P increases by accretion from the cloud at a rate which at any time is mkv, where m is the mass and v the speed of the particle, k being a constant. Show that, after P has fallen a distance x,

$$kv^2 = g(1 - e^{-2kx})$$

and find the distance the particle has fallen after a time t.

5. Let $\underline{H}$ be the angular momentum about a fixed point O, of a system of particles in motion. Show that $\frac{d\underline{H}}{dt} = \underline{M}$, where $\underline{M}$ is the total moment about O of the external forces acting on the system.

A uniform circular disc, centre C, of mass m and radius r can rotate in a vertical plane about a smooth horizontal axis perpendicular to its plane through a point A on its rim. Initially, the disc is held at rest with AC horizontal. It is then released. Find the components of the force on the axis when AC makes an angle θ with the downward vertical.

Also calculate the magnitude of the force on the axis in the following cases

(a) when AC is vertical,

(b) when $\theta = \frac{\pi}{2}$ and

(c) when θ is greatest.

6. For a body in motion about an axis, show that the impulsive moment of the resultant force about the axis is equal to the gain of angular momentum.

A uniform rod AB of length $2a$ and mass $3m$ is at rest on a smooth, horizontal table and the rod is free to rotate on the table about a smooth, vertical axis through end A. A particle P of mass $\frac{3m}{5}$ moving at u ms^{-1} on the table at right angles to AB strikes the rod at C where $AC = \frac{5a}{4}$. P adheres to the rod. Calculate the angular speed with which the rod begins to rotate.