THE OPEN UNIVERSITY OF SRI LANKA

BACHELOR OF TECHNOLOGY HONORS IN ENGINEERING /

BACHELOR OF SOFTWARE ENGINEERING HONORS – LEVEL 04

FINAL EXAMINATION – 2015/2016

MPZ4140 / MPZ4160 - DISCRETE MATHEMATICS I

DURATION – THREE HOURS

Date: 05th December 2016 **Time: 0930hrs – 1230 hrs.**

Instructions:

- **Answer only six questions.**
- **Please answer a total of six questions choosing at least one from each section A, B, and C.**
- **State any assumption that you made.**
- **All symbols are in standard notation.**

SECTION – A

01. i. Define a statement.
Let the universe of discourse be the set of all integers. Determine the truth values of the following statements.
a) $(\forall x)p(x)$ where $p(x)$: $(x^2 \geq 0)$
b) $(\forall x)q(x)$ where $q(x)$: $(x^2 - 5x + 6 = 0)$
c) $(\exists x)r(x)$ where $r(x)$: $(x^2 - 5x + 6 = 0)$ **[30%]**

ii. Write down each of the following statements in terms of **p**, **q** and **r**, and logical connectives, where **p**: I am awake; **q**: I work hard; and **r**: I dream of home.
a) I am awake implies that I work hard.
b) If I dream of home, then I am awake and I work hard.
c) I am not awake if and only if I dream of home or I work hard..
d) I do not work hard only if I am awake and I do not dream of home. **[20%]**

iii. Let p, q, and r be three statements.
Verify that $[(p \vee q) \wedge (p \rightarrow r)] \rightarrow (q \vee r)$ is a tautology or not. **[20%]**

iv. State the "converse" and "contrapositive" of each of the following statement: **[20%]**
a) If I like Logic, then I will study.
b) If it snows. then he does not drive the car.

v. Show that $(p \wedge q) \vee \sim p \equiv \sim p \vee q$, by using laws of the algebra of propositions. **[10%]**

02. i. Express the following arguments into symbolic form and test the validity by using truth table: **[30%]**

If I was reading the news paper in the kitchen, then my glasses are on the kitchen table.
When my glasses are on the kitchen table, I see my glasses.
I did not see my glasses.
Therefore glasses are not on the kitchen table and I was not reading the news paper in the kitchen.

ii. Prove that $\forall a \in \mathbb{R}, \exists b \in \mathbb{R}$ such that $a^2 + b = 5$. **[20%]**

iii. Write down the negation of the following statements:

a) If the teacher is absent, then some students do not complete their homework.
b) All the students completed their homework and the teacher is present.
c) Some of the students did not complete their homework or the teacher is absent **[30%]**

iv. Prove De Morgant's laws for propositions by using truth tables. **[20%]**

03. i. Using mathematical induction, for a positive integer n, prove each of the following:

a) $\dfrac{x^{n+1}-1}{x-1} = 1 + x + x^2 + \cdots + x^n$, **[30%]**

b) $2\sum_{r=1}^{n} r = n(n+1)$. **[30%]**

ii. Give an indirect proof for the followings:
for any real number x, if $x^3 + 2x + 33 \neq 0$, then $x + 3 \neq 0$. **[20%]**

iii. Find a counter example for the following statement is true; **[10%]**
$\exists n \in \mathbb{N},\ 7n + 1$ and $7n - 1$ are not primes.

iv. Disprove $\forall a, b \in \mathbb{R}$, if $a > b$, then $a^2 > b^2$. **[10%]**

SECTION – B

04. i. Determine which of the following sets are finite: **[20%]**

a). A = {month in year}, b). B = {positive integer lees than 1}
c). C = {odd integer}, d). D = {positive integer divisors of 12}

ii. Giving reasons describe the following sets:
a) $\emptyset$ b) $\{0\}$ c) $\{\emptyset\}$. **[15%]**

Tern Over

iii. Let P, Q, and R be three sets. Without using Venn diagram, show that $P \cap (Q \cup R) = (P \cap Q) \cup (P \cap R)$. **[30%]**

iv. Let $U = \mathbb{N} = \{1, 2, 3, ...\}$ be the universal set. Let $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6, 7\}$, $and\ D = \{2, 4, 6, ...\}$. Find the all elements in each of the following set: A^C, B^C, D^C, $A \oplus B$, $A \oplus D$. **[35%]**

05. i. Define the Cartesian product sets $A \times B$ of the sets A and B.
a). Let $A = \{00, 01, 10, 11\}$, and $B = \{0, 1\}$. Find $A \times B$, A^2.
b). Find a, and b such that $(3a + b, 2a - b) = (11, -1)$. **[20%]**

ii. Let R be a relation on A={1, 2, 3, 4} defined by R= {(1,1), (2, 2), (2, 3), (3, 2), (4, 2), (4, 4)}. Show that R is neither reflexive, nor transitive. **[10%]**

iii. Let $\mathbb{N}$ be the set of all natural numbers. The relation R_1 on the set $\mathbb{N} \times \mathbb{N}$ of ordered pairs of natural numbers is defined as:
$$(a, b)R_1(c, d) \Leftrightarrow a - c = b - d.$$
Prove that R_1 is an equivalence relation. **[40%]**

iv. Prove that the following relation R_2 on the set of positive integer is a partial order.
$$R_2 = \{(x, y): x \mid y\}.$$
[30%]

06. i. Show that $f: \mathbb{R} \to \mathbb{R}$ given by $f(x) = 8x - 4$ is a one to one and onto function. Find the inverse function of f. **[20%]**

ii. Define $h: \mathbb{Z} \to \mathbb{Z}$ by $h(n) = 3n + 5$. Show that h is not a onto function. **[10%]**

iii. Prove that $f: \mathbb{N} \to \mathbb{N}$, where $f(x) = x^2$ is a one to one function, but $g: \mathbb{Z} \to \mathbb{Z}_0^+$ where $g(x) = x^2$ is not a one to one function. **[30%]**

iv. Find the domain D of each of the following function:
a) $f(x) = x^2 + 2x - 15$,
b) $g(x) = \sqrt{64 - x^2}$,
c) $h(x) = 3/(x^2 - 6x + 8)$. **[20%]**

iv. Let $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ be functions defined by $f(x) = 3x^2 + 2x$ and $g(x) = 5x + 2$ for all $x \in \mathbb{R}$ respectively. Find the, $f \circ g$, and $g \circ f$. **[20%]**

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SECTION – C

07. i. Given integers a, b, c and d, show that,

a) If $a|b$, and $a > 0, and\ b > 0$, then $a \leq b$. [20%]

b) If $a|b$ and $b|c$, then $a|c$. [10%]

c) if $a|b$, $a|c$, and $a|d$, then $a|(b - 2c)$ and $a|(2b - 5c + 3d)$. [20%]

d) If $a|b$ and $a|c$, then $a^2|b(c - b)$. [20%]

ii. Prove that if $n \in \mathbb{Z}^+$ and $(3n + 1)|(n^2 - 3n + 4)$, then
$(3n^2 + 4n + 1)|(2n^3 - 4n^2 + 2n + 8)$. [20%]

iii. Show that if n is positive even integer, then $4^n - 1$ is not a prime. [10%]

08. i. Let $a, b, c, d \in \mathbb{Z}$. Show that;

a). if $\gcd(a, b) = 1$, then $\gcd(a + b, a - b) = 1$ or 2, [30%]

b). if $\gcd(a, b) = d$, then $\gcd(a/d\ , b/d) = 1$, [15%]

c). if $a|c$ and $b|c$, with $\gcd(a, b) = 1$, then $ab|c$. [25%]

ii. Find the $gcd(12378, 3054)$, and express it as $12378\,x + 3054\,y$ by using the Euclidean Algorithm, and determine integers x_0 and y_0 of the following equation:
$$12378\,x_0 + 3054\,y_0 = 30.$$ [30%]

09.

i. Let $a \equiv b(mod\ n)$ and $c > 0$.
Show that $a + c \equiv b + c(mod\ n)$ and $ac \equiv bc(mod\ n)$ [25%]

ii. If $x \equiv y(mod\ m)$, then show that $y \equiv x(mod\ m)$. [10%]

iii. Solve the following system of congruence:

$17x \equiv 3(mod\ 2)$
$17x \equiv 3(mod\ 3)$
$17x \equiv 3(mod\ 5)$
$17x \equiv 3(mod\ 7)$

[65%]