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THE OPEN UNIVERSITY OF SRI LANKA
B.Sc./B.Ed. Degree Programme, Continuing Education Programme
APPLIED MATHEMATICS – LEVEL 05
AMU 3189/ AME 5189- STATISTICS II
FINAL EXAMINATION - 2010/2011

Duration: Two Hours.

Date: 19.08.2011	**Time: 9.30 a.m. – 11.30 a.m**

Non programmable calculators are permitted. Statistical tables are provided.

Answer FOUR questions only.

(1) Let $X_1, X_2, X_3, \ldots, X_n$ be a random sample from a uniform distribution with density given by

$$f(x;\theta) = \frac{1}{5-\theta} \quad ; \theta \le x \le 5$$

(i) Find the mean of the above distribution

(ii) Derive a moment estimator for θ. Is the moment estimator derived by you unbiased for θ? Prove your answer.

(iii) Derive the maximum likelihood estimator for θ.

(iv) A sample drawn from the above distribution is given below. Calculate the moment estimate and the maximum likelihood estimate based on the estimators derived by you in parts (ii) and (iii).

4.23	3.79	3.00	4.35	4.46	4.86	3.87	3.91	4.69	3.86
3.60	4.36	3.01	4.55	4.47	4.06	4.88	3.71	3.59	4.88

(2) The random variable X denotes the lifetime of a certain type of battery. Suppose X follows a normal distribution with mean μ and variance σ^2 so that the probability density function of X is given by $f(x,\mu,\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-1}{2}\left[\frac{(x-\mu)}{\sigma}\right]^2}$; $-\infty < x < \infty$

Let $X_1, X_2, X_3, \ldots, X_n$ denote lifetimes of n randomly chosen batteries from the above population.

a) Using the Factorization Theorem show that $\sum_{i=1}^{n} X_i$ and $\sum_{i=1}^{n} X_i^2$ are jointly sufficient for μ and σ^2.

b) Using the part (a) find UMVU estimators for μ and σ^2.

c) Find the Cramer-Rao lower bound (CRLB) for the variance of an unbiased estimator for 2μ.

(3) A certain manufacturing company produces circular metal disks. The diameter of the disk is used as the dimension of the product. To maintain the quality of the product random samples of 25 disks are chosen in every two hours. If the production process is in control, it is known that diameter of a disk is normally distributed with mean 5cm and standard deviation 0. 02cm. If the mean diameter of disk is not 5cm or the standard deviation of the diameter of disk is greater than 0.02cm, the production process is considered as out of control. The following table gives some statistics which were calculated from a sample collected on a particular day at 2.00 pm.

Descriptive Statistics: Diameter

Variable	Count	Mean	StDev	Minimum	Median	Maximum
Diameter	25	5.0050	0.0190	4.9988	4.9998	5.0033

(i) Construct a 95% confidence interval for mean diameter of the discs produced by the process. Round off your answer to four decimal places. Interpret your result.

(ii) Construct a 95% confidence interval for variance of the diameters of the discs produced by the process. Round off your answer to four decimal places. Interpret your result.

(iii) Was the production process in control at 2.00 p.m. on the particular day? Justify your answer.

(4) A random variable X is distributed as $N(\mu, 16)$. Suppose we know that μ can take only one of the two values 2 and 4. A random sample $X_1, X_2, X_3, \ldots, X_{20}$ is drawn from the distribution to test the simple null hypothesis $H_0 : \mu = 2$ against the simple alternative hypothesis $H_1 : \mu = 4$.

(i) Using the Neyman- Pearson Lemma show that the best critical region of significance level 0.025 to test the above hypothesis is given by $\bar{X} \geq 3.75$.

(ii) The following table gives you a random sample of size 20 drawn from the above distribution. Test the above mentioned hypotheses and give your conclusion at 0.025 significance level.

-1.78	2.46	0.58	3.12	1.7
-0.27	5.6	5.84	8.37	9.73
-2.38	0.36	-1.36	3.13	7.32
7.29	-2.43	3.3	-0.94	0.11

(iii) Calculate the power of the test.

(5) (a) A boy used to play a certain game with a six sided dice. One day he found some problem with the dice and assumed that the dice was not fair. To test his assumption he threw the dice 60 times and the no of occurrences of the scores were recorded. Following table illustrates the results. Test whether the assumption made by the boy is correct at 5% level of significance. Clearly state your conclusion.

Score	1	2	3	4	5	6
Frequency	8	10	6	12	13	11

(b) Using the context of the test done in the part (a) briefly explain the following terms.

i. Significance level
ii. Critical region
iii. Most powerful test

(6)

A biologist weights each individual mouse in a random sample consisting of ten mice and records weight to the nearest gram. The mice are then fed on a special diet and after 15 days each mouse is weighted again. The weight to the nearest gram is recorded. The results as follows.

Initial weight	50	49	48	52	40	43	51	46	41	42
Weight after 15 days	52	50	50	55	42	45	52	48	42	44

(i) Assuming that the biologist has recorded the results in random order in both occasions and the variance of the initial weight and the variance of the weight after 15 days are equal, examine the possibility that there has been a significant increase in mean weight over 15 days. Use 5% level of significance.

(ii) If the biologist has recorded the results in the same order in both occasions, explain how this fact would alter your analysis of these data. (Without calculation)

(iii) Suppose you are asked to consult the biologist to conduct this experiment. Which method you suggest given in part (i) and part (ii) to biologist to carry on. Give reasons for your answer.

(iv) Clearly state all the assumptions that you have made in part(i), part(ii) and part(iii).