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The Open University of Sri Lanka
B.Sc/B.Ed Degree Programme
Final Examination - 2010/2011
Applied Mathematics – Level 5
AMU3186/AME5186 – Quantum Mechanics
Duration :- Two hours

Date:- 09.07.2011 **Time:- 1.30 p.m.-3.30 p.m.**

Answer **Four** Questions Only.

1. (a) Show that, in the Compton effect, the electron recoil angle ϕ and the photon scattering angle θ satisfy

$$\tan\phi = \frac{\cot \theta/2}{1 + E/mc^2}$$

where E is the initial energy of the photon and the m is the rest mass of the electron.

(b) An X-ray photon of wave length 0.1mm is incident on a stationary electron. If the photon scattering angle is 30^0, find the electron recoil angle. You are given that,

$c = 3\times10^8\, ms^{-1}$ $m = 9.108\times10^{-31}\, kg$ and $h = 6.626\times10^{-34}\, Js$

2. Consider a particle described by a Gaussian wave packet, given in the usual notation by

$$\psi(x) = A\exp\left[\frac{-(x+x_0)^2}{8a^2}\right] \text{ ; where } x_0 \text{ and } a(\neq 0) \text{ are constants.}$$

(a) If ψ is normalized calculate A.
(b) Calculate $\langle \Delta x \rangle$, in the usual notation.

You may use the result $\int_{-\infty}^{+\infty} e^{-\alpha^2 y^2}\, dy = \frac{\sqrt{\pi}}{\alpha}$; where α is a non zero constant.

3. (a) Define Eigen values and Eigen functions of an operator.

Show that $u(x)=e^{-x^2/2}$ is an eigen function of the operator $\hat{A}\left[x,\frac{d}{dx}\right]=\frac{d^2}{dx^2}-x^2$ and find the corresponding eigen value.

(b) Use Mathematical Induction to, show that $\left[\hat{x}^n,\hat{p}_x\right]=in\hbar x^{n-1}$, when n is a positive integer.

(c) If $\hat{H}=\frac{\hat{p}_x^2}{2m}+\frac{1}{2}\omega^2\hat{x}^2$, then evaluate the following commutators.

(i) $\left[\hat{x},\hat{H}\right]$ (ii) $\left[\hat{H},\hat{p}_x\right]$

where $\hat{p}_x$ is momentum operator and $\hat{H}$ is the Hamiltonian operator.

4. If $\hat{A}$ is an operator corresponding to a quantum observable and $\langle\hat{A}\rangle$ is the corresponding expectation value, show that $\frac{d\langle\hat{A}\rangle}{dt}=\frac{1}{ih}\left\langle\left[\hat{A},\hat{H}\right]\right\rangle+\left\langle\frac{\partial\hat{A}}{\partial t}\right\rangle$, where $\hat{H}$ is the time independent Hamiltonian operator for the quantum system.

Prove that,

(i) $\frac{d}{dt}\langle\hat{r}\rangle=\frac{1}{m}\langle\hat{p}_x\rangle$ (ii) $\frac{d}{dt}\langle\hat{p}_x\rangle=\langle-\nabla\hat{V}\rangle$

When $F=-\nabla V$, show that $\langle F\rangle=m\frac{d^2}{dt^2}\langle r\rangle$.

5. A particle of mass m moves in the negative x direction, under a potential defined by,

$$V(x)=\begin{cases}V_0\,; x<0\\ 0\,; x\geq 0\end{cases}$$

(i) If the energy of the particle is $E(>V_0)$, find the wave function $\psi(x)$ for $x\geq 0$ and $x<0$.

(ii) Find the transmission coefficient T.

(iii) Comment on the behavior on the transmission coefficient for each of the cases $E >> V_0$ and $E \simeq V_0$.

6. Angular momentum of a particle is defined as a vector $\underline{L}$, given by $\underline{L} = \underline{r} \times \underline{p}$ where $\underline{p}$ is the momentum and $\underline{r}$ is the position vector of the particle w.r.t a fixed origin O. Write down the Cartesian components $\hat{L}_x, \hat{L}_y, \hat{L}_z$ of the angular momentum operator. Hence obtain the angular momentum operator in polar coordinates (r, θ, ϕ).

Prove the following relations for the angular momentum operator.

(i) $\left[\hat{L}_x, \hat{L}_y\right] = i\hbar\hat{L}_z$ (ii) $\left[\hat{H}, \hat{L}_z\right] = 0$