The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination – 2010/2011
Level 05-Applied Mathematics
AMU 3185/AME 5185 – EM Theory & Special Relativity
**Duration :- Two and Half Hours**

**Date :- 07.07.2011** **Time :- 1.30 p.m. – 3.30 pm.**

**Answer Four Questions Only.**

01. An imaginary open surface $S$ is in the form of a spherical cap $r = a$, $0 \le \theta \le \alpha$, $0 \le \omega \le 2\pi$, where $(r, \theta, \omega)$ denote spherical polar coordinates. Define the flux of a vector $\underline{E}$ through $S$.

    (a) Use your definition to calculate the electric flux through $S$ if a uniform electric field $\underline{E}$ acting parallel to the axis of symmetry.
    (b) What is the flux if $\underline{E}$ acts perpendicular to the axis of symmetry?
    (c) Determine the flux through $S$ if a point charge $Q$ is placed at the centre $(r = 0)$. Deduce the flux for the above case when $\alpha = \pi$.

02. If $\underline{A}$ satisfies the equations

$$div\underline{A} = 0, \qquad \nabla^2 \underline{A} = \frac{1}{c^2}\ddot{\underline{A}}$$

and $\underline{E}$ and $\underline{H}$ are defined by the relations

$$\underline{E} = -\frac{1}{c}\dot{\underline{A}}, \qquad \underline{H} = curl\,\underline{A}\ ,$$

show that $\underline{E}$ and $\underline{H}$ satisfy the Maxwell's equations,

$$curl\,\underline{H} - \frac{1}{c}\dot{\underline{E}} = 0, \qquad div\,\underline{H} = 0 \quad ;$$
$$curl\,\underline{E} + \frac{1}{c}\dot{\underline{H}} = 0, \qquad div\,\underline{E} = 0$$

for the electromagnetic field in vacuuo. (Dots denote partial differentiation with respect to time)

Show that $\underline{A} = \underline{i}\,a\cos\frac{2\pi}{\lambda}(z - ct) + \underline{j}\,a\sin\frac{2\pi}{\lambda}(z - ct)$, where $a$ and $\lambda$ are constants, is a possible solution.

03. (a) Explain briefly the following terms:
(i) Electrical potential (ii) Potential difference
(b) Show that the potential $V_r$ at a point distance $r$ from a point charge $Q$ is given by

$V_r = \frac{1}{4\pi\varepsilon}\frac{Q}{r}$, where $\varepsilon$ is the permitivity of the medium.

(c) A charge $2q$ is uniformly distributed inside an insulating material in the form of a sphere of radius $r$. The permitivity of the material is $\varepsilon_0$.

Show that the potential at a distance $\frac{r}{2}$ from the centre is $\frac{11q}{16\pi\varepsilon_0 r}$.

04. Point charges $q, -q'$ and $-q'$ are placed at points $O(0,0), A(a,0)$ and $B(-a,0)$ respectively, $q$ and $q'$ being both positive.
(i) Given that $q > 2q'$, show that the extreme line of force ending on $A$ is issued from $O$ making an angle $\alpha$ with $OA$, where $\alpha = 2\sin^{-1}\sqrt{\frac{q'}{q}}$.
(ii) Given that $q < 2q'$, show that the extreme line of force ending on $A$ is issued from $O$ making an angle $\beta$ with $OA$ produced, where $\beta = 2\cos^{-1}\sqrt{\frac{q}{2q'}}$.

05. (a) A circular loop carries a current $I$. Derive an expression for the magnetic field at any point on the line passing through its centre and perpendicular to its plane.

(b) Derive an expression for the magnetic field at a point on the axis of a solenoid of radius R and N turns/metre, which carries a current I.

06. Derive the Loreznt transformation equations.
Verify that the above equations can be expressed in the form

$$x' = x\cosh\alpha - ct\sinh\alpha$$
$$y' = y$$
$$z' = z$$
$$ct' = ct\cosh\alpha - x\sinh\alpha$$

where $\tanh\alpha = v/c$

Deduce that

$$x' - ct' = (x - ct)e^{\alpha}$$
$$x' + ct' = (x + ct)e^{\alpha}$$