The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination – 2010/2011
Level 05-Applied Mathematics
AMU3181/AME5181 – Fluid Mechanics

Duration :- Two Hours

Date :- 22.06.2011 **Time :- 9.30 a.m. – 11.30 a. m.**

Answer any Four Questions.

01. In usual notation, show that the pressure gradient due to gravity $\frac{dp}{dz}$ is given by,

$$\frac{dp}{dz} = -\rho g.$$

The temperature in the earth's atmosphere, at rest, remains constant and the atmospheric pressure varies with the density according to Boyle's law $p = k\rho$, where k is a constant. The acceleration due to gravity at a height z above the ground level is given by $g = \frac{g_0 a^2}{(a+z)^2}$, where a is the radius of the earth and g_0 is the value of g at ground level.

Show that the pressure p at a height z is given by $p = p_0 e^{\frac{-g_0 az}{k(a+z)}}$, where p_0 is the value of p at ground level.

02. Velocity, in cylindrical polar coordinates (r,θ,z), at a point in a fluid is given by $\underline{q} = \frac{m}{r} e_r$, where m is a constant. Verify that this motion is a possible one, and determine the streamlines.

Also, show that the motion is irrotational with velocity potential $\phi = -m \log r$, and that the streamlines cut equipotentials orthogonally.

03. Two equal 2–D sources, each of strength m, are placed at points $A(a,0)$ and $B(-a,0)$, in unbounded incompressible fluid. Write down the velocity potential and stream function for the combined flow.

Also, show that

(i) the streamlines are given by $x^2 - y^2 + \lambda xy = a^2$, where λ is a parameter.

(ii) the x–axis is a streamline, and find the velocity at any point there.

(iii) at any point on the circle with AB as a diameter, fluid velocity is parallel to the y- axis and its magnitude is inversely proportional to $|y|$.

04. With the usual notation, assuming the Euler's equation

$$\frac{\partial U}{\partial t}+\nabla\left(\frac{1}{2}U^2\right)-U\wedge(\nabla\wedge U)=F-\frac{1}{\rho}\nabla P,$$

show that $\frac{P}{\rho}+\frac{1}{2}U^2-\Omega=$ Constant along the stream lines. State any assumptions you make.

A liquid of constant density ρ, contained in a vessel rotates under gravity, with uniform angular velocity ω about the *OZ-axis* (pointing vertically upwards). Show that the pressure at a point having cylindrical polar coordinates (r,θ,z) is of the form $p=p_0+\frac{\rho}{2}\omega^2r^2-\rho gz$, where p_0 is a constant (to be identified).

05. Assume the drag force *F*, exerted on a body is a function of the fluid density ρ, fluid viscosity μ, diameter d and velocity q.
Show that the drag force can be expressed as $F=d^2q^2\rho\phi(\text{Re})$ where ϕ is some unknown function and *Re* is the Reynold's number.

A prototype boat propeller has a diameter of $1.0m$. It is necessary to determine the force it will experience when water flows past at $5m/s$. A model propeller is available in diameter $0.1m$ and can be placed in a wind tunnel. To obtain dynamically similar conditions, at what velocity would the air need to flow in the wind tunnel?

$\mu_{\text{water}}=1.0\times10^{-3}\,kg/ms$ $\qquad$ $\mu_{\text{air}}=1.7\times10^{-5}\,kg/ms$

$\rho_{\text{water}}=1000kg/m^3$ $\qquad$ $\rho_{\text{air}}=12.5kg/m^3$

06. The velocity vector $\underline{q}$ in a motion of a perfect incompressible fluid of density ρ is given as

$$\underline{q}=\begin{cases}\omega r\underline{e}_\theta, & 0\le r<a\\ \dfrac{\omega a^2}{r}\underline{e}_\theta, & r\ge a,\end{cases}$$

Referring to cylindrical polar coordinates (r,θ,z), where ω is a positive constant, find the **vorticity vector** and identify the type of motion, in each region.
If the fluid motion takes place under no external forces and pressure at infinity is p_∞, use Euler's equation of motion to find the pressure distribution in the outer region $(r>a)$. Also, show that the pressure in the inner region is $p_\infty+\frac{\rho}{2}\omega^2\left(r^2-2a^2\right)$.