MPZ 3230 – Assignment # 01

Academic Year - 2006

01. (i) ABCD is a parallelogram. M is the midpoint of AB. AC and DM cross each other at the point N. To when ratio point N divides AC and DM?

(ii) ABC is a triangle and θ is a angle between two sides of triangle. Then show that cosine rule by using triangle.

(iii) Let $\underline{a} = \underline{i} + \underline{j} + \underline{k}$, $\underline{b} = -\underline{i} + \underline{j}$ and $\underline{c} = 3\underline{i} + \underline{k}$. Find the cosine of the angle between the following vectors (a) $\underline{a}$, $\underline{b} + \underline{c}$ (b) $\underline{a} + \underline{c}$, $\underline{a} - \underline{c}$ by using cosine rule. Are that the $\underline{a}, \underline{b}, \underline{c}$ vectors coplaner or not? Whether the $\underline{a}, \underline{b}, \underline{c}$ vectors are orthogonal? Define orthonormal vector.

(iv) The 3 vectors are given in standard notation as, $\underline{x} = (1, 1, 0)$, $\underline{y} = (0, 2, 0)$ and $\underline{z} = (0, 1, 1)$ using these vectors verify the following vector relation, $\underline{a} \cdot (\underline{b} \times \underline{c}) = (\underline{d})^2$. Where $\underline{a} = \underline{x} \times \underline{y}$, $\underline{b} = \underline{y} \times \underline{z}$, $\underline{c} = \underline{z} \times \underline{x}$, $\underline{d} = \underline{x} \cdot (\underline{y} \times \underline{z})$ Interpret your results geometrically.

02. (a) Show that $\frac{d}{dt}(\underline{a}.\underline{b}\times\underline{c}) = \frac{d\underline{a}}{dt}.\ \underline{b}\times\underline{c} + \underline{a}.\ \frac{d\underline{b}}{dt}.\times\underline{c} + \underline{a}.\underline{b}\times\frac{d\underline{c}}{dt}$

(b) If $\underline{r} = (\text{cosnt})\underline{i} + (\text{Sin nt})\,\underline{j}$; where n is a constant and t varies, find

(i) $\underline{r}\times\frac{d\underline{r}}{dt}$ (ii) $\underline{r}.\frac{d\underline{r}}{dt}$ (iii) $\frac{d^2\underline{r}}{dt^2}$ (iv) $\left|\frac{d\underline{r}}{dt}\right|$ (v) $\left|\frac{d^2\underline{r}}{dt^2}\right|$

(c) Let the position vector of a moving particle be $\underline{r}(t) = t^2\,\underline{i} - 2t\,\underline{j} + (t^2 + 2t)\,\underline{k}$,where t represents time.

(i) Show that the particle goes through the point (4, -4, 8). At what time does it do this?

(ii) Find the velocity vector and the speed of the particle at time t, at the time when it passes through the point (4, -4, 8).

03. (a) Solve the following differential equations.

(i) $\frac{dy}{dx} = (y - 4x)^2$

(ii) $(y - xy)^2 = (x + x^2y)\frac{dy}{dx}$

(iii) $(5x^4 + 3x^2y^2 - 2xy^3)\,dx + (2x^3y - 3x^2y^2 - 5y^4)\,dy = 0$

(iv) $(x^2 + 3xy + y^2)\,dx - x^2\,dy = 0$ (v) $\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$

04. (i)Find the solution of the boundry value problem.

$$\frac{dy}{dx} = \frac{y \cos x}{1+2y^2} \quad ; \quad y(0)=1$$

(ii) A body falling vertically under gravity encounters resistance of the atmosphere. If the resistance varies as the velocity. Assuring that the equation of motion is given by $\frac{du}{dt} = g - ku$, . where u is the velocity, k is a constant and g is the acceleration due to gravity.

(i) Find the general solution of the equation.
(ii) If t increases very large then find the value of U.
(iii) If $U = \frac{dx}{dt}$ where x is the distance fallen by the body from rest in time t.

Show that $x = \frac{gt}{k} - \frac{g}{k^2}(1-e^{-kt})$

05. Determine whether or not each of equations exact. If it is exact or not find the solution by using suitable method.

(i) $(2x + 4y) + (2x - 2y)\frac{dy}{dx} = 0$

(ii) $(x^2y + y+1)\ dx + x(1+x^2)\ dy = 0$

(iii) $\frac{dy}{dx} - \frac{3y}{x+1} = (x+1)^4$

(iv) $\operatorname{Cos} x \frac{dy}{dx} - y \operatorname{Sin} x = x^2$

(v) $\tan x \frac{dy}{dx} + y = e^x \tan x$

Please send your assignment on or before **30.06.2006** to the following address. Please send your answer with your address (write back of your answer sheet) and when you are doing assignment use both sides of paper.

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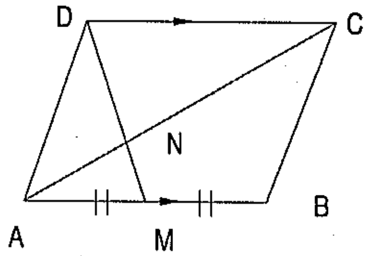
MPZ 3230 - Model Answer – 01

Academic Year – 2006

(01). (i)



Let $\overrightarrow{AD} = \underline{a}$, $\overrightarrow{DC} = \underline{b}$ Then $\overrightarrow{AC} = \underline{a} + \underline{b}$ $\overrightarrow{DM} = \frac{\underline{b}}{2} - \underline{a}$

If $\overrightarrow{DN} = \lambda\overrightarrow{DM}$ and $\overrightarrow{AN} = \mu\overrightarrow{AC}$ Then $\overrightarrow{AD} = \overrightarrow{AN} + \overrightarrow{ND}$

$\underline{a} = \mu\overrightarrow{AC} + \left(-\lambda\overrightarrow{DM}\right) = \mu(\underline{a} + \underline{b}) - \lambda\left(\frac{\underline{b}}{2} - \underline{a}\right) = (\mu + \lambda)\underline{a} + \left(\mu - \frac{\lambda}{2}\right)\underline{b}$

But $\underline{a}$ & $\underline{b}$ are not parallel. Therefore $\mu + \lambda = 1$ & $\mu - \frac{\lambda}{2} = 0$

$$\lambda = \frac{2}{3}, \ \mu = \frac{1}{3}$$

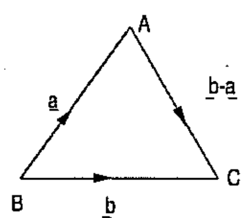
$\overrightarrow{DN} = \lambda\overrightarrow{DM}$ & $\overrightarrow{AN} = \mu\overrightarrow{AC}$

$\overrightarrow{DN} = \frac{2}{3}\overrightarrow{DM}$ $\overrightarrow{AN} = \frac{1}{3}\overrightarrow{AC}$

$\frac{\overrightarrow{DN}}{\overrightarrow{DM}} = \frac{2}{3}$ $\frac{\overrightarrow{AN}}{AC} = \frac{1}{3}$

$\overrightarrow{DN} : \overrightarrow{DM} = 2 : 3$ $\overrightarrow{AN} : \overrightarrow{AC} = 1 : 3$ Therefore DN : NM = 2:1 & AN : NC = 1:2

(ii)



Let $\overrightarrow{BA} = \underline{a}$ and $\overrightarrow{BC} = \underline{b}$. Then $\overrightarrow{AC} = \underline{b} - \underline{a}$; $\overrightarrow{CA} = \underline{a} - \underline{b}$

But $|-\underline{a} + \underline{b}| = |\underline{a} - \underline{b}|$

$|\underline{a} - \underline{b}|^2 = |\underline{a}|^2 + |\underline{b}|^2 - 2|\underline{a}||\underline{b}|\cos\theta$

$(\underline{a} - \underline{b}).(\underline{a} - \underline{b}) = \underline{a}.\underline{a} + \underline{b}.\underline{b} - 2|\underline{a}||\underline{b}|\cos\theta$

$\underline{a}.\underline{a} - \underline{a}.\underline{b} - \underline{b}.\underline{a} + \underline{b}.\underline{b} = \underline{a}.\underline{a} + \underline{b}.\underline{b} - 2|\underline{a}||\underline{b}|\cos\theta$

$$\underline{a}.\underline{b} = |\underline{a}||\underline{b}|\cos\theta,$$

$\cos\theta = \dfrac{\underline{a}.\underline{b}}{|\underline{a}||\underline{b}|}$ θ is angle between vector $\overline{BA}$ and $\overline{BC}$

Cosine rule is proved.

(iii) Let $\underline{a} = \underline{i} + \underline{j} + \underline{k}$, $\underline{b} = -\underline{i} + \underline{j}$, $\underline{c} = 3\underline{i} + \underline{k}$

(a) $\underline{b} + \underline{c} = 2\underline{i} + \underline{j} + \underline{k}$, θ - angle between $\underline{a}$ and $\underline{b} + \underline{c}$

$$\therefore \cos\theta = \frac{\underline{a}.(\underline{b} + \underline{c})}{|\underline{a}||\underline{b} + \underline{c}|} = \frac{2+1+1}{\sqrt{3}\sqrt{6}} = \frac{4}{3\sqrt{2}} = \frac{2\sqrt{2}}{3}$$

$$\theta = \cos^{-1}\frac{2\sqrt{2}}{3}$$

(b) $\underline{a} + \underline{c} = 4\underline{i} + \underline{j} + 2\underline{k}$, $\underline{a} - \underline{c} = -2\underline{i} + \underline{j}$, θ-angle between $\underline{a} + \underline{c}$ and $\underline{a} - \underline{c}$

$$\cos\theta = \frac{(\underline{a} + \underline{c}).(\underline{a} - \underline{c})}{|\underline{a} + \underline{c}||\underline{a} - \underline{c}|} = \frac{(4\underline{i} + \underline{j} + 2\underline{k}).(-2\underline{i} + \underline{j})}{\sqrt{21}\sqrt{5}} = \frac{-8+1}{\sqrt{105}} = \frac{-7}{\sqrt{105}}$$

$$\theta = \cos^{-1}\left(\frac{-7}{\sqrt{105}}\right)$$

$\underline{a} \times \underline{b}.\underline{c} = 0$ iff $\underline{a}, \underline{b}, \underline{c}$ are coplanar.

$\underline{a} \times \underline{b} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 1 & 1 \\ -1 & 1 & 0 \end{vmatrix} = -\underline{i} - \underline{j} + 2\underline{k}$, $\quad \underline{a} \times \underline{b}.\underline{c} = (-\underline{i} - \underline{j} + 2\underline{k}).(3\underline{i} + \underline{k}) = -3 + 2 = -1 \neq 0$

$\therefore \underline{a}, \underline{b}, \underline{c}$ are not coplanar.

$\underline{a}.\underline{b} = 0$ iff $\underline{a}$ and $\underline{b}$ are orthogonal

$\underline{a}.\underline{b} = (\underline{i} + \underline{j} + \underline{k}).(-\underline{i} + \underline{j}) = 0$. Therefore $\underline{a}$ and $\underline{b}$ are orthogonal.

$\underline{a}.\underline{c} = (\underline{i} + \underline{j} + \underline{k}).(3\underline{i} + \underline{k}) = 4 \neq 0$, Therefore $\underline{a}$ and $\underline{c}$ are not orthogonal.

$\underline{b}.\underline{c} = (-\underline{i} + \underline{j}).(3\underline{i} + \underline{k}) = -3 \neq 0$, Therefore $\underline{b}$ and $\underline{c}$ are not orthogonal.

Definition of orthonormal vector

If a set of vectors is such that the magnitude of each one of them is unity and any two of them are orthogonal, then they are called on orthonormal set of vectors.

(iv) $\underline{x}=(1,1,0)$, $\underline{y}=(0,2,0)$, $\underline{z}=(0,1,1)$

$\underline{a}=\underline{x}\times\underline{y}$ $\quad$ $\underline{b}=\underline{y}\times\underline{z}$ $\quad$ $\underline{c}=\underline{z}\times\underline{x}$

$$=\begin{vmatrix}\underline{i} & \underline{j} & \underline{k}\\ 1 & 1 & 0\\ 0 & 2 & 0\end{vmatrix} \qquad =\begin{vmatrix}\underline{i} & \underline{j} & \underline{k}\\ 0 & 2 & 0\\ 0 & 1 & 1\end{vmatrix} \qquad =\begin{vmatrix}\underline{i} & \underline{j} & \underline{k}\\ 0 & 1 & 1\\ 1 & 1 & 0\end{vmatrix}$$

$\underline{a}=2\underline{k}$ $\quad$ $=2\underline{i}$ $\quad$ $=-\underline{i}+\underline{j}-\underline{k}$

$\underline{d}=\underline{x}.(\underline{y}\times\underline{z})=(1,1,0).(2,0,0)=2$, $(\underline{d})^2=4$

$$\underline{b}\times\underline{c}=\begin{vmatrix}\underline{i} & \underline{j} & \underline{k}\\ 2 & 0 & 0\\ -1 & 1 & -1\end{vmatrix}=2\underline{j}+2\underline{k}$$

$\underline{a}.(\underline{b}\times\underline{c})=(0,0,2).(0,2,2)=4$ $\qquad$ Therefore $\underline{a}.(\underline{b}\times\underline{c})=(\underline{d})^2$

The volume of the parallelepiped formed by the 3 vectors $\underline{a},\underline{b},\underline{c}$ is equal to the square of the magnitude of the vector $\underline{d}$.

(02) (a) $$\frac{d}{dt}(\underline{a}.\underline{b}\times\underline{c})=\underline{a}.\frac{d}{dt}(\underline{b}\times\underline{c})+\underline{b}\times\underline{c}.\frac{d}{dt}\underline{a}$$

$$=\underline{a}.\left(\underline{b}\times\frac{d\underline{c}}{dt}+\frac{d\underline{b}}{dt}\times\underline{c}\right)+\underline{b}\times\underline{c}.\frac{d\underline{a}}{dt}$$

$$=\underline{a}.\underline{b}\times\frac{d\underline{c}}{dt}+\underline{a}.\frac{d\underline{b}}{dt}\times\underline{c}+\frac{d\underline{a}}{dt}.\underline{b}\times\underline{c}$$

(b) $\underline{r}=\cos nt\underline{i}+\sin nt\underline{j}$

$$\frac{d\underline{r}}{dt}=-n\sin nt\underline{i}+n\cos nt\underline{j}=n(-\sin nt\underline{i}+\cos nt\underline{j})$$

$$\frac{d^2\underline{r}}{dt^2}=n^2(-\cos nt\underline{i}-\sin nt\underline{j})$$

(i) $$\underline{r}\times\frac{d\underline{r}}{dt}=\begin{vmatrix}\underline{i} & \underline{j} & \underline{k}\\ \cos nt & \sin nt & 0\\ -n\sin nt & n\cos nt & 0\end{vmatrix}=n\underline{k}$$

(ii) $\underline{r}.\frac{d\underline{r}}{dt} = -n\sin nt.\cos nt + n\cos nt \sin nt = 0$

(iii) $\frac{d^2\underline{r}}{dt^2} = n^2(-\cos nt\underline{i} - \sin nt\,\underline{j}) = -n^2\underline{r}$

(iv) $\left|\frac{d\underline{r}}{dt}\right| = n$

(v) $\left|\frac{d^2\underline{r}}{dt^2}\right| = n^2$

(c) $\underline{r}(t) = t^2\underline{i} - 2t\underline{j} + (t^2 + 2t)\underline{k}$

(i) If the particle goes through the point (4, -4, 8) then $\underline{r}(t) = (4,-4,8)$.

$\therefore t^2 = 4 \Rightarrow t = \pm 2$

$-2t = -4 \Rightarrow t = 2$

$t^2 + 2t = 8 \Rightarrow t^2 + 2t - 8 = 0$

$(t+4)(t-2) = 0$

$t = -4$ or $t = 2$

t = 2 is satisfy the above three equations. Therefore the particle goes through the point (4, -4, 8) at t = 2.

(ii) $\frac{d\underline{r}}{dt} = 2t\underline{i} + -2\underline{j} + (2t+2)\underline{k}$

Velocity vector at time $t = 2t\underline{i} - 2\underline{j} + (2t+2)\underline{k}$

Speed at time $t = \sqrt{4t^2 + 4 + (2t+2)^2}$

$= \sqrt{8(t^2 + t + 1)}$

At the time t = 2 when it passes through the point (4, -4, 8)

Velocity vector at time $t = 2; \Rightarrow 4\underline{i} - 2\underline{j} + 6\underline{k}$

Speed at time $t = 2; \Rightarrow \sqrt{56}$

(03) (i) $\frac{dy}{dx} = (y - 4x)^2$

Let $z = y - 4x$

$\frac{dz}{dx} = \frac{dy}{dx} - 4$

$\frac{dz}{dx} = z^2 - 4$

$$\frac{1}{z^2-4}dz = dx$$

$$\frac{1}{z^2-4} = \frac{1}{4}.\frac{1}{(z-2)} - \frac{1}{4}.\frac{1}{(z+2)}$$

$$\int\frac{dz}{z^2-4} = \frac{1}{4}\int\frac{dz}{z-2} - \frac{1}{4}\int\frac{dz}{z+2} = \int dx$$

$$\frac{1}{4}\ln\left|\frac{z-2}{z+2}\right| = x + c; \text{ c is a constant.}$$

$$\frac{1}{4}\ln\left|\frac{y-4x-2}{y-4x+2}\right| = x + c$$

Correct question (03) (ii) as follows

(ii) $$(y-xy)^2 = (x+xy)^2\frac{dy}{dx}$$

$$\frac{(1-x)^2}{x^2}dx = \frac{(1+y)^2dy}{y^2}$$

$$\int\left(\frac{1}{x^2} - \frac{2}{x} + 1\right)dx = \int\left(\frac{1}{y^2} + \frac{2}{y} + 1\right)dy$$

$$-\frac{1}{x} - 2\ln x + x = -\frac{1}{y} + 2\ln y + y + c$$

OR

$$\left(y - xy^2\right) = \left(x + x^2y\right)\frac{dy}{dx}$$

$$y(1-xy) = x(1+xy)\frac{dy}{dx}$$

Let $v = xy$ $$\frac{dv}{dx} = x\frac{dy}{dx} + y$$

$$\frac{dy}{dx} = \frac{\frac{dv}{dx} - \frac{v}{x}}{x}$$

$$\frac{v}{x}(1-v) = \frac{x(1+v)}{x}\left(\frac{dv}{dx} - \frac{v}{x}\right)$$

$$\frac{v}{x}(1-v) = (1+v)\left(\frac{dv}{dx} - \frac{v}{x}\right)$$

$$(1+v)\frac{dv}{dx}=\frac{v}{x}(1-v+1+v)$$

$$(1+v)\frac{dv}{dx}=\frac{2v}{x}$$

$$\int\frac{(1+v)dv}{v}=\int\frac{2dx}{x}$$

$$\ln v+v=2\ln x+c$$

$$\ln xy+xy=2\ln x+c$$

(iii) $$\left(5x^4+3x^2y^2-2xy^3\right)dx+\left(2x^3y-3x^2y^2-5y^4\right)dy=0$$

Let $y=vx$

$$\frac{dy}{dx}=v+x.\frac{dv}{dx}$$

$$\left(5x^4+3v^2x^4-2v^3x^4\right)=\left(5v^4x^4+3v^2x^4-2vx^4\right)\left(v+x\frac{dv}{dx}\right)$$

$$x\frac{dv}{dx}=\frac{5+3v^2-2v^3-5v^5-3v^3+2v^2}{5v^4+3v^2-2v}$$

$$\int\frac{5v^4+3v^2-2v}{-5\left(v^5+v^3-v^2-1\right)}dv=\int\frac{1}{x}dx$$

$$-\frac{1}{5}\ln\left|v^5+v^3-v^2-1\right|=\ln|x|+\ln|c|$$

$$-\frac{1}{5}\ln\left|\frac{y^5}{x^5}+\frac{y^3}{x^3}-\frac{y^2}{x^2}-1\right|=\ln|x|+\ln|c|$$

$$y^5+x^2y^3-y^2x^3-x^5=c$$

(iv) $$\left(x^2+3xy+y^2\right)dx-x^2dy=0$$

Let $y=vx$ $\qquad \frac{dy}{dx}=v+x.\frac{dv}{dx}$

$$v+x\frac{dv}{dx}=\frac{x^2+3vx^2+v^2x^2}{x^2}$$

$$x\frac{dv}{dx}=v^2+3v+1-v$$

$$\int\frac{dv}{(v+1)^2}=\int\frac{dx}{x}$$

$$-\frac{1}{(1+v)} = \ln x + c \quad \Rightarrow \quad -\frac{1}{1+\frac{y}{x}} = \ln x + c$$

$$-\frac{x}{x+y} = \ln x + c$$

(v) $\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2}$

$y = vx \qquad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$v + x\frac{dv}{dx} = \frac{x^2+vx^2+v^2x^2}{x^2}$$

$$x\frac{dv}{dx} = 1+v^2$$

$$\int\frac{dv}{1+v^2} = \int\frac{dx}{x}$$

$$\tan^{-1} v = \ln x + c$$

$$\tan^{-1}\frac{y}{x} = \ln x + c$$

(04) (i) $\frac{dy}{dx} = \frac{y\cos x}{1+2y^2}; y(0) = 1$

$$\int\left(\frac{1+2y^2}{y}\right)dy = \int\cos x dx$$

$$\ln y + y^2 = \sin x + c$$

Since $y(0) = 1$; $\ln 1 + 1 = 0 + c$

$c = 1$

$\therefore$ solution : $\ln y + y^2 = \sin x + 1$

(ii) $\frac{du}{dt} = g - ku$

(i) $\int\frac{du}{g-ku} = \int dt$

$$-\frac{1}{k}\ln|g-ku| = t + c$$

General solution: $\frac{1}{g-ku} = e^{kt+kc}$; c is an arbitrary constant.

(ii) Since t increases very large ;

Then $t \to \infty$

$e^{kt+kc} \to \infty$

$\therefore \frac{1}{g-ku} \to \infty$

Therefore $g - ku = 0$ $\therefore u = \frac{g}{k}$

(iii) If $u = \frac{dx}{dt}$

In the rest, $t = 0$ and $x = 0$

Then $\frac{dx}{dt} = u = 0$

$\therefore \frac{1}{g-k.0} = e^{0+kc}$; $\frac{1}{g} = e^{kc}$; $g = e^{-kc}$

$g - k.\frac{dx}{dt} = e^{-k(t+c)}$

$k\frac{dx}{dt} = g - e^{-k(t+c)}$

$k\int dx = \int\left[g - e^{-k(t+c)}\right]dt$

$k[x]_0^x = \left[gt - \frac{e^{-k(t+c)}}{(-k)}\right]_0^t$

$kx = gt + \frac{e^{-(t+c)k}}{k} - \frac{1}{k}e^{-kc}$

Since $g = e^{-kc}$

$\therefore kx = gt + \frac{1}{k}ge^{-tk} - \frac{1}{k}g$

$x = \frac{gt}{k} - \frac{g}{k^2}\left(1 - e^{-kt}\right)$

(05) (i) $(2x+4y) + (2x-2y)\frac{dy}{dx} = 0$

$f(x,y) = 2x + 4y$ and $g(x,y) = 2x - 2y$

$\frac{\partial f}{\partial y} = 4$ $\frac{\partial g}{\partial x} = 2$

$\frac{\partial f}{\partial y} \neq \frac{\partial g}{\partial x}$ $\therefore$ This is not an exact equation.

But we can find the solution by using suitable substitution,

Let $y = vx$; $\frac{dy}{dx} = v + x\frac{dv}{dx}$

$$(2x+4vx)+(2x-2vx)\left(v+x\frac{dv}{dx}\right)=0$$

$$\int\frac{dx}{x} = \int\frac{1-v}{v^2-3v-1}dv$$

$$\int\frac{dx}{x} = -\frac{1}{2}\int\frac{2v-3}{v^2-3v-1}dv - \frac{1}{2}\int\frac{dv}{\left(v-\frac{3}{2}\right)^2-\left(\frac{\sqrt{13}}{2}\right)^2}$$

$$\ln x = -\frac{1}{2}\ln\left(v^2-3v-1\right)+\frac{1}{\sqrt{13}}\left|\ln(\operatorname{cosec}\theta-\cot\theta)\right|+c$$

$$\ln x = -\frac{1}{2}\ln\left(\frac{y^2}{x^2}-\frac{3y}{x}-1\right)+\frac{1}{\sqrt{13}}\ln\frac{2\left(\frac{y}{x}\right)-3-\sqrt{13}}{2\sqrt{\left(\frac{y}{x}\right)^2-3\left(\frac{y}{x}\right)-1}}$$

$$\frac{\sqrt{13}}{2}\sec\theta = v-\frac{3}{2} \Rightarrow \sec\theta = \frac{2v-3}{\sqrt{13}}$$

$$\frac{\sqrt{13}}{2}\sec\theta\tan\theta d\theta = dv \quad ; \tan\theta = \sqrt{\frac{(2v-3)^2}{13}-1} = \sqrt{\frac{4v^2-12v-8}{13}}$$

(ii) $\left(x^2y+y+1\right)dx + x\left(1+x^2\right)dy = 0$

$f(x,y) = x^2y+y+1$ and $g(x,y) = x\left(1+x^2\right)$

$\frac{\partial f}{\partial y} = x^2+1$ $\qquad$ $\frac{\partial g}{\partial x} = 1+3x^2$

$\frac{\partial f}{\partial y} \neq \frac{\partial g}{\partial x}$ $\therefore$ This equation is not an exact.

$$\frac{1}{g}\left(\frac{\partial f}{\partial y}-\frac{\partial g}{\partial x}\right) = \frac{x^2+1-1-3x^2}{x\left(1+x^2\right)} = \frac{-2x}{1+x^2}$$

The integrating factor $p(x) = e^{-\int\frac{2x}{1+x^2}dx}$ $\therefore p(x) = e^{-\ln\left(1+x^2\right)} = \frac{1}{\left(1+x^2\right)}$

Multiplying by the integrating factor,

$$x\frac{dy}{dx} = -\frac{(x^2y + y + 1)}{x^2+1} \quad \Rightarrow \quad xdy + ydx + \frac{1}{x^2+1}dx = 0$$

$xy + \tan^{-1} x = c$; c is an arbitrary constant

(iii) $\frac{dy}{dx} - \frac{3y}{x+1} = (x+1)^4$

$(x+1)\ \frac{dy}{dx} - 3y = (x+1)^5$

$f(x, y) = -3y$ and $g(x, y) = (x+1)$

$\frac{\partial f}{\partial y} = -3$ $\qquad$ $\frac{\partial g}{\partial x} = 1$

$$\frac{\partial f}{\partial y} \neq \frac{\partial g}{\partial x}$$

∴ This is not an exact equation.

$$\frac{1}{g}\left(\frac{\partial f}{\partial y} - \frac{\partial g}{\partial x}\right) = \frac{-3-1}{(x+1)} = -\frac{4}{(x+1)}$$

Integrating factor $p(x) = e^{-\int\frac{4}{(x+1)}dx} = \frac{1}{(x+1)^4}$

Multiplying by the integrating factor,

$$\frac{1}{(x+1)^3}\frac{dy}{dx} - \frac{3y}{(x+1)^4} = (x+1)$$

$$d\left[\frac{4}{(x+1)^3}\right] = (x+1)$$

$$\frac{y}{(x+1)^3} = \int(x+1)dx$$

$$y = (x+1)^3\left(\frac{x^2}{2} + x\right) + c;$$ c is an arbitrary constant.

(iv) $\cos x\frac{dy}{dx} - y\sin x = x^2$

$f(x, y) = -y\sin x$ and $g(x, y) = \cos x$

$\frac{\partial f}{\partial y} = -\sin x$ $\qquad$ $\frac{\partial g}{\partial x} = -\sin x$

$$\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$$

$\therefore$ This is an exact equation.

$$u = \int -y \sin x dx + F(y) = y \cos x + F(y)$$

$$\frac{\partial u}{\partial y} = g(x, y)$$

$$\cos x + \frac{\partial F}{\partial y} = \cos x \quad \Rightarrow \quad \frac{\partial F}{\partial y} = 0$$

$F(y) = c$; where c is an arbitrary constant

$$\frac{d}{dx}(y \cos x + c) = x^2$$

$y \cos x = \frac{x^3}{3} + c$; where c is an arbitrary constant

(v) $\tan x \frac{dy}{dx} + y = e^x \tan x$

$f(x, y) = y$ and $f(x, y) = \tan x$

$\frac{\partial f}{\partial y} = 1$ and $\frac{\partial g}{\partial x} = \sec^2 x$

$\frac{\partial f}{\partial y} \neq \frac{\partial g}{\partial x}$; $\therefore$ This is not an exact equation.

$$\frac{1}{g}\left(\frac{\partial f}{\partial y} - \frac{\partial g}{\partial x}\right) = \frac{1 - \sec^2 x}{\tan x} = -\tan x$$

The integrating factor $p(x) = e^{-\int \tan x dx} = \cos x$

Multiplying by integrating factor,

$$\sin x \frac{dy}{dx} + \cos x . y = e^x \sin x$$

$$d(\sin xy) = e^x \sin x$$

$$\sin xy = \int e^x \sin x dx$$

$$y \sin x = \frac{e^x}{2}(\sin x - \cos x) + c$$

Where c is an arbitrary constant.

$$\frac{\partial f}{\partial y} = \frac{\partial g}{\partial x}$$

$\therefore$ This is an exact equation.

$$u = \int -y\sin x dx + F(y) = y\cos x + F(y)$$

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Where c is an arbitrary constant.