MPZ 3230 – Assignment # 02
Academic Year – 2006

(1) Obtain the general solution of

(a) $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=0$

(b) $\frac{d^2y}{dx^2}+3\frac{dy}{dx}-4y=10\cos x$

(c) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=2x^2+3$

(2) (a) Using variation of parameter method, obtain the general solution of

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}-8y=x^2e^{3x}$$

(b) Using Trial Functions method, obtain the general solution of

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=e^{2x}\left(2-x^2\right)$$

(c) Solve the boundary value problem

$$\frac{d^2y}{dt^2}+2\frac{dy}{dt}+y=e^t \quad \text{subject to } y(0)=0 \text{ and } y'(0)=1$$

(3) (a) Using the definition of the Laplace transform, find the Laplace transform of the following functions.

(i) $f(x)=x^2$

(ii) $f(x)=\sin^2 ax$

(iii) $f(x-c)=\begin{cases}0 & ;x<c\\ 1 & ;x\geq c\end{cases}$

(b) Find the inverse Laplace transforms of the following functions.

(i) $\frac{1-3s}{2s^2-7}$ (ii) $\frac{s-3}{s^2+4s+6}$ (iii) $\frac{6s-4}{s^2-4s+20}$

(c) Solve the following boundary value problems using the Laplace Transform method.

(i) $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=4e^{2x};\ y(0)=-3,\ y'(0)=5$

(ii) $y''(t)+9y(t)=\cos 2t;\ y(0)=c_1$ and $y'(0)=c_2$

(4) (a) Show that the matrix

$$\begin{pmatrix}-1 & 3 & 2\\ 0 & 5 & 4\\ 1 & -3 & 1\end{pmatrix}$$

Can be expressed in the sum of symmetric Hermition and skew symmetric Hermition matrix.

(b) Determine the values of α, β, γ when

$$\begin{pmatrix} 0 & 2\beta & \gamma \\ \alpha & \beta & -\gamma \\ \alpha & -\beta & \gamma \end{pmatrix} \text{ is orthogonal.}$$

(5) (a) Suppose A is diagonal and B is triangular such as,

$$A = \begin{pmatrix} a_1 & 0 & 0 \\ 0 & a_2 & 0 \\ 0 & 0 & a_3 \end{pmatrix} \text{ and } B = \begin{pmatrix} b_1 & b_2 & b_3 \\ 0 & b_4 & b_5 \\ 0 & 0 & b_6 \end{pmatrix}$$

(i) Show that adj A is diagonal and adj B is triangular.

(ii) Find determinant of A and determinant of B.

(b) (i) Without expanding show that,

$$\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} = (a-b)(b-c)(c-a)(a+b+c)$$

(iii) find the value of $\begin{vmatrix} a & b & b & b \\ b & a & b & b \\ b & b & a & b \\ b & b & b & a \end{vmatrix}$

(6) (a) Find the inverse of matrix

$$\begin{pmatrix} 1 & 1 & 2 \\ 3 & 2 & 4 \\ 0 & 1 & -2 \end{pmatrix}$$ by using the method of elementary row transformations.

(b) (i) Given that $z = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$

Prove that $\frac{\partial^2 z}{\partial y \partial x} = \frac{x^2 - y^2}{x^2 + y^2}$

(ii) If $z = \log_e(x^2 + y^2)$ show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

(iii) State associative property for multiplication (3 variables) & then prove it based on truth table .

Please send your assignment **<u>on or before 15.08.2006</u>** to the following address.

<u>Please send your answer with your address (write back of your answer sheet) and when you are doing assignment use both sides of paper.</u>

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MPZ 3230 Model Answer # 02
Academic Year – 2006

(1) (a) $\frac{d^2y}{dx^2}+6\frac{dy}{dx}+9y=0$

The characteristic equation is,

$m^2+6m+9=0$

$\therefore m=-3$ (twice)

Hence the general solution is,

$y=Ae^{-3x}+Bxe^{-3x}$; where A & B are constants

(b) $\frac{d^2y}{dx^2}+3\frac{dy}{dx}-4y=10\cos x$

The roots of the characteristic equation,

$m^2+3m-4=0$ are m = -4 and m = 1

The complementary function is $y=Ae^{-4x}+Be^{x}$; A & B are constants

Particular integral, $y_p=\frac{1}{(D+4)(D-1)}10\cos x$

$$y_p=-\frac{10\cos x}{5(D+4)}+\frac{10\cos x}{5(D-1)}$$

$$=-2\left(e^{-4x}\int e^{4x}\cos x\,dx\right)+2\left(e^{x}\int e^{-x}\cos x\,dx\right)$$

$$=-2e^{-4x}\left[\frac{e^{4x}}{17}(4\cos x+\sin x)\right]+2e^{x}e^{-x}\frac{(\sin x-\cos x)}{2}$$

$$=\frac{15}{17}\sin x-\frac{25}{17}\cos x$$

General solution

$$y=Ae^{-4x}+Be^{x}+\frac{15}{17}\sin x-\frac{25}{17}\cos x$$

(c) $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+2y=2x^2+3$

The roots of the characteristic equation.

$$m^2+2m+2=0 \text{ are } m_1=-1+i \text{ and } m_2=-1-i$$

The complementary function is

$y_c = e^{-x}(A\cos x + B\sin x)$ Where A & B are constants

Particular Integral,

Using trial function method,

$y_p = \propto_0 x^2 + \propto_1 x + \propto_2$

$y_p' = 2\propto_0 x + \propto_1$

$y_p'' = 2\propto_0$

$\therefore y_p'' + 2y_p' + 2y_p = 2\propto_0 + 2(2\propto_0 x + \propto_1) + 2(\propto_0 x^2 + \propto_1 x + \propto_2)$

$= 2\propto_0 x^2 + 2(\propto_1 + 2\propto_0)x + 2(\propto_0 + \propto_1 + \propto_2)$

Thus $y = \propto_0 x^2 + \propto_1 x + \propto_2$ is a particular integral of $y'' + 2y' + 2y = 2x^2 + 3$

If $2\propto_0 = 2$, $2(\propto_1 + 2\propto_0) = 0$ and $2(\propto_0 + \propto_1 + \propto_2) = 3$

$\Rightarrow \propto_0 = 1, \ \propto_1 = -2, \ \propto_2 = \frac{5}{2}$

$\therefore y_p = x^2 - 2x + \frac{5}{2}$

$\therefore$ general solution:

$y = e^{-x}(A\cos x + B\sin x) + x^2 - 2x + \frac{5}{2}$

(2) (a) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 8y = x^2e^{3x}$

Since the characteristic equation is,

$(m^2 + 2m - 8) = (m+4)(m-2) = 0 \Rightarrow$ m= - 4 and m=2

The complementary functions are,

$y_1 = e^{-4x}$ and $y_2 = e^{2x}$

$y_c = Ae^{-4x} + Be^{2x}$; where A & B are constants

The wranstian $\omega(x) = y_1'y_2 - y_2'y_1$

$= -4e^{-4x}e^{2x} - 2e^{2x}e^{-4x}$

$= -6e^{-2x}$

$$\int \frac{y_2 f(x)}{\omega(x)}dx = \int \frac{e^{2x}x^2e^{3x}}{-6e^{-2x}}dx = -\frac{e^{7x}}{6}\left(\frac{x^2}{7} - \frac{2x}{7^2} + \frac{2}{7^3}\right)$$

$$\int \frac{y_1 f(x)}{\omega(x)} dx = \int \frac{e^{-4x} x^2 e^{3x}}{-6e^{-2x}} dx = -\frac{e^x}{6}(x^2 - 2x + 2)$$

$$y_p(x) = -e^{-4x} \frac{e^{7x}}{6}\left(\frac{x^2}{7} - \frac{2x}{7^2} + \frac{2}{7^3}\right) + e^{2x}.\frac{e^x}{6}(x^2 - 2x + 2)$$

$$= \frac{e^{3x}}{6}\left(\frac{x^2}{7} - \frac{16x}{7^2} + \frac{114}{343}\right)$$

General solution:

$$y = Ae^{-4x} + Be^{2x} + e^{3x}\left(\frac{x^2}{7} - \frac{16x}{49} + \frac{114}{343}\right)$$

(b)Find the only complementary function.

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x}(2 - x^2)$$

The characteristic equation is,

$m^2 - 4m + 4 = (m-2)(m-2) = 0 \Rightarrow$ m=2(twice)

$\therefore$ The complementary function is,

$y_c = Ae^{2x} + Bxe^{2x}$, where A and B are arbitrary constants.

(c) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + y = e^t$ subject to $y(0) = 0$ and $y'(0) = 1$

The characteristic equation is,

$$m^2 + 2m + 1 = (m+1)(m+1) = 0 \Rightarrow m = -1 \text{ (twice)}$$

$\therefore$ The complementary function is,

$y_c = Ae^{-t} + Bte^{-t}$: where A and B are arbitrary constants

By using trial function,

Assume $y_p = \propto e^t$

$$y_p' = \propto e^t$$

$$y'' = \propto e^t$$

Therefore $\propto e^t + 2 \propto e^t + \propto e^t = e^t$

$$4 \propto e^t = e^t$$

$$\therefore \propto = \frac{1}{4}$$

(ii) $f(x) = x^4 - x - 10$

$f(1) = 1^4 - 1 - 10 = -10 < 0$

$f(2) = 2^4 - 2 - 10 = 4 > 0$

$\therefore$According to the intermediate value theorem there exist a root between 1 & 2.But a root is closed to 2.

Therefore $x_0 = 2$.

By using Newton-Raphson method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad f'(x) = 4x^3 - 1$$

When $n = 0$; $x_0 = 2$

$$f(x_0) = f(2) = 4, \ f'(x_0) = 4.2^3 - 1 = 31$$

$$x_1 = 2 - \frac{4}{31} = 1.8710$$

$$|x_1 - x_0| = |1.8710 - 2| = 0.1290$$

When $n = 1$; $x_1 = 1.8710$

$$f(x_1) = (1.8710)^4 - 1.8710 - 10 = 0.3835$$

$$f'(x_1) = 4(1.8710)^3 - 1 = 25.1988$$

$$x_2 = 1.8710 - \frac{0.3835}{25.1988}$$

$$= 1.8558$$

$$|x_2 - x_1| = |1.8558 - 1.8710| = 0.0152$$

When $n = 2$, $x_2 = 1.8558$

$$f(x_2) = (1.8558)^4 - (1.8558) - 10$$
$$= 0.0053$$

$$f'(x_2) = 4(1.8558)^3 - 1 = 24.5654$$

$$x_3 = 1.8558 - \frac{0.0053}{24.5654} = 1.8556$$

$$|x_3 - x_2| = |1.8556 - 1.8558| = 0.0002$$

Real root of f(x) = 1.8556

ween I

$$= \frac{1}{2s} - \frac{s}{2(s^2+4a^2)} = \frac{2a^2}{s(s^2+4a^2)}$$

$$L\{\sin^2 ax\} = \frac{2a^2}{s(s^2+4a^2)} \quad \text{for } s>0$$

(iii) $f(x-c) = \begin{cases} 0 & : \quad x<c \\ 1 & ; \quad x \geq c \end{cases}$

$$L\{f(x-c)\} = \int_0^\infty e^{-sx} f(x-c)\,dx$$

$$= \int_0^c e^{-sx}(0)\,dx + \int_c^\infty e^{-sx}(1)\,dx$$

$$= \int_c^\infty e^{-sx}\,dx$$

$$= \left[\frac{e^{-sx}}{-s}\right]_c^\infty$$

$$= \frac{1}{s}e^{-sc} \quad \text{for } s>0$$

(b) (i) $L^{-1}\left\{\frac{1-3s}{2s^2-7}\right\} = L^{-1}\left\{\frac{1}{2\left(s^2-\frac{7}{2}\right)}\right\} - \frac{3}{2}L^{-1}\left\{\frac{s}{\left(s^2-\frac{7}{2}\right)}\right\}$

$$= \frac{1}{2\sqrt{\frac{7}{2}}}\sinh\sqrt{\frac{7}{2}}t - \frac{3}{2}\cosh\sqrt{\frac{7}{2}}t$$

(ii) $\frac{s-3}{s^2+4s+6} = \frac{s+2}{(s+2)^2+2} - \frac{5}{(s+2)^2+2}$

$$L^{-1}\left\{\frac{s-3}{s^2+4s+6}\right\} = L^{-1}\left\{\frac{s+2}{(s+2)^2+2}\right\} - 5L^{-1}\left\{\frac{1}{(s+2)^2+2}\right\}$$

$$= e^{-2t}\cos\sqrt{2}t - \frac{5}{\sqrt{2}}e^{-2t}\sin\sqrt{2}t$$

(iii) $\frac{6s-4}{s^2-4s+20} = \frac{6(s-2)}{(s-2)^2+4^2} + \frac{8}{(s-2)^2+4^2}$

By Gauss-seidel method,

$$x_1^{(m+1)} = 1 + x_2^{(m)} - x_3^{(m)} + 10x_4^{(m)}$$

$$x_2^{(m+1)} = 2x_1^{(m+1)} + 4x_3^{(m)} - 21x_4^{(m)} + 3$$

$$x_3^{(m+1)} = \frac{1}{6}\left\{-x_1^{(m+1)} - x_2^{(m+1)} + 13x_4^{(m)} - 11\right\}$$

$$x_4^{(m+1)} = \frac{1}{19}\left\{-9 + 2x_1^{(m+1)} - 3x_2^{(m+1)} - x_3^{(m+1)}\right\}$$

n	x_1	x_2	x_3	x_4
0	0	0	0	0
1	1	5	$-\frac{17}{6} = -2.83$	$-\frac{115}{114} = -1.0088$
2	-1.26	10.349	-5.534	-1.949
3	-2.607	16.579	-8.385	-2.924

$x_1 = -2.61$
$x_2 = 16.58$
$x_3 = -8.38$
$x_4 = -2.92$

(ii) Jacobi Method,
We can write the system as matrix form

$$\underbrace{\begin{pmatrix} -1 & 1 & -1 & 10 \\ 2 & -1 & 4 & -21 \\ -1 & -1 & -6 & 13 \\ -2 & 3 & 1 & 19 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}}_{\underline{x}} = \underbrace{\begin{pmatrix} -1 \\ -3 \\ 11 \\ -9 \end{pmatrix}}_{\underline{b}}$$

Then $\underline{x}^{(k+1)} = D^{-1}\underline{b} - D^{-1}(L+u)\underline{x}^{(k)}$

$$D = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -6 & 0 \\ 0 & 0 & 0 & 19 \end{pmatrix}$$

(4) (a) If a is any square matrix, A + A* is a symmetric Hermition matrix and A – A* is skew symmetric Hermition matrix

$$\frac{1}{i}(A - A^*) = -i(A - A^*)$$

Therefore $\frac{1}{i}(A - A^*)$ is skew symmetric Hermition matrix.

We can show that $A = P + i(Q)$

Where $P = \frac{1}{2}(A + A^*)$ is symmetric Hermition matrix and

$Q = \frac{1}{2i}(A - A^*)$ is skew symmetric Hermition matrix.

$$A = \begin{pmatrix} -1 & 3 & 2 \\ 0 & 5 & 4 \\ 1 & -3 & 1 \end{pmatrix} \quad A^* = (\overline{A})^t = \begin{pmatrix} -1 & 0 & 1 \\ 3 & 5 & -3 \\ 2 & 4 & 1 \end{pmatrix}$$

$$P = \frac{1}{2}(A + A^*) = \frac{1}{2}\begin{pmatrix} -2 & 3 & 3 \\ 3 & 10 & 1 \\ 3 & 1 & 2 \end{pmatrix}$$

$$Q = \frac{1}{2i}(A - A^*) = \frac{1}{2i}\begin{pmatrix} 0 & 3 & 1 \\ -3 & 0 & 7 \\ -1 & -7 & 0 \end{pmatrix} \Rightarrow iQ = \frac{1}{2}\begin{pmatrix} 0 & 3 & 1 \\ -3 & 0 & 7 \\ -1 & -7 & 0 \end{pmatrix}$$

$$\left[\frac{1}{2}(A + A^*)\right]^* = \frac{1}{2}\begin{pmatrix} -2 & 3 & 3 \\ 3 & 10 & 1 \\ 3 & 1 & 2 \end{pmatrix}$$

Since $\frac{1}{2}(A + A^*) = \left[\frac{1}{2}(A + A^*)\right]^*$ therefore $\frac{1}{2}(A + A^*)$ is symmetric Hermition matrix

If $iQ = (a_{ij})_{3\times 3}$ & $a_{ij} = -\overline{a_{ji}}$ for all i,j.

Therefore $\frac{i}{2i}(A - A^*)$ is skew symmetric Hermition matrix

Therefore we can write matrix A as the sum of symmetric Hermition and skew symmetric Hermition matrix.

(b) If $AA^t = I$ then A is called orthogonal matrix.

2nd iteration

$$x^{(2)}=\begin{pmatrix}-1&0&0&0\\0&-1&0&0\\0&0&-\frac{1}{6}&0\\0&0&0&\frac{1}{19}\end{pmatrix}\begin{pmatrix}-1\\-3\\11\\-9\end{pmatrix}-\begin{pmatrix}-1&0&0&0\\0&-1&0&0\\0&0&-\frac{1}{6}&0\\0&0&0&\frac{1}{19}\end{pmatrix}\begin{pmatrix}0&1&-1&10\\2&0&4&-21\\-1&-1&0&13\\-2&3&1&0\end{pmatrix}\begin{pmatrix}1\\3\\-\frac{11}{6}\\-\frac{9}{19}\end{pmatrix}$$

$$=\begin{pmatrix}1\\3\\-\frac{11}{6}\\-\frac{9}{19}\end{pmatrix}-\begin{pmatrix}-\frac{11}{114}\\-\frac{263}{57}\\\frac{193}{114}\\\frac{31}{114}\end{pmatrix}=\begin{pmatrix}\frac{125}{114}\\\frac{434}{57}\\\frac{-67}{19}\\\frac{-85}{114}\end{pmatrix}$$

3rd iteration

$$x^{(3)}=\begin{pmatrix}-1&0&0&0\\0&-1&0&0\\0&0&\frac{-1}{6}&0\\0&0&0&\frac{1}{19}\end{pmatrix}\begin{pmatrix}-1\\-3\\11\\-9\end{pmatrix}-\begin{pmatrix}-1&0&0&0\\0&-1&0&0\\0&0&\frac{-1}{6}&0\\0&0&0&\frac{1}{19}\end{pmatrix}\begin{pmatrix}0&1&-1&10\\2&0&4&-21\\-1&-1&0&13\\-2&3&1&0\end{pmatrix}\begin{pmatrix}\frac{125}{114}\\\frac{434}{57}\\\frac{-67}{19}\\\frac{-85}{114}\end{pmatrix}$$

$$=\begin{pmatrix}1\\3\\\frac{-11}{6}\\\frac{-9}{19}\end{pmatrix}-\begin{pmatrix}\frac{-70}{19}\\\frac{-427}{114}\\\frac{1049}{342}\\\frac{976}{1083}\end{pmatrix}=\begin{pmatrix}\frac{89}{19}\\\frac{769}{114}\\\frac{-838}{171}\\\frac{-1489}{1083}\end{pmatrix}=\begin{pmatrix}4.684\\6.746\\-4.901\\-1.372\end{pmatrix}$$

Solution:

$$\begin{pmatrix}x_1^{(3)}\\x_2^{(3)}\\x_3^{(3)}\\x_4^{(3)}\end{pmatrix}=\begin{pmatrix}4.68\\6.75\\-4.9\\-1.37\end{pmatrix}$$

$$\det B = \begin{vmatrix} b_1 & b_2 & b_3 \\ 0 & b_4 & b_5 \\ 0 & 0 & b_6 \end{vmatrix} = b_1(b_4 b_6) - b_2(0) + b_3(0) = b_1 b_4 b_6$$

(b) (i)

$$\begin{vmatrix} 1 & a & a^3 \\ 1 & b & b^3 \\ 1 & c & c^3 \end{vmatrix} \xrightarrow[c_2 \to c_2 - c_3]{c_1 \to c_1 - c_2} \begin{vmatrix} a & a-b & (a-b)(a^2+ab+b^2) \\ a & b-c & (b-c)(b^2+bc+c^2) \\ 1 & c & c^3 \end{vmatrix}$$

$$= (a-b)(b-c)\begin{vmatrix} 0 & 1 & a^2+ab+b^2 \\ 0 & 1 & b^2+bc+c^2 \\ 1 & c & c^3 \end{vmatrix}$$

$$= (a-b)(b-c)(b^2+bc+c^2-a^2-ab-b^2)$$

$$= (a-b)(b-c)(c-a)(a+b+c)$$

(ii)

$$\begin{vmatrix} a & b & b & b \\ b & a & b & b \\ b & b & a & b \\ b & b & b & a \end{vmatrix} \xrightarrow{r_1 \to r_1 + r_2 + r_3 + r_4} = (a+3b)\begin{vmatrix} 1 & 1 & 1 & 1 \\ b & a & b & b \\ b & b & a & b \\ b & b & b & a \end{vmatrix}$$

$$\xrightarrow[\substack{r_2 \to r_2 - r_3 \\ r_3 \to r_3 - r_4}]{r_1 \to r_1 - r_2} = (a+3b)\begin{vmatrix} 0 & 0 & 0 & 1 \\ b-a & a-b & 0 & b \\ 0 & b-a & a-b & b \\ 0 & 0 & b-a & a \end{vmatrix}$$

$$= (a+3b)\begin{vmatrix} b-a & a-b & 0 \\ 0 & b-a & a-b \\ 0 & 0 & b-a \end{vmatrix}$$

$$= (a-b)(a+3b)\begin{vmatrix} -1 & 1 & 0 \\ 0 & b-a & a-b \\ 0 & 0 & b-a \end{vmatrix}$$

$$= (a-b)(a-b)^2(a+3b)\begin{vmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= (a-b)^3(a+3b)$$

$$= a^4 + 2ab^3 - 6a^2b^2 - 3b^4$$

(6) (a) Using elementary row transformations,

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 2 & 4 \\ 0 & 1 & -2 \end{pmatrix} \quad \text{we know that } A = I.A$$

$$\begin{pmatrix} 1 & 1 & 2 \\ 3 & 2 & 4 \\ 0 & 1 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.A \quad \underline{R_2 \to R_2 - 3R_1}$$

$$\begin{pmatrix} 1 & 1 & 2 \\ 0 & -1 & -2 \\ 0 & 1 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.A \quad \underline{R_2 \to -R_2}$$

$$\begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 1 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 3 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.A \quad \underset{R_3 \to R_3 - R_2}{\underline{R_1 \to R_1 - R_2}}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & -4 \end{pmatrix} = \begin{pmatrix} -2 & 1 & 0 \\ 3 & -1 & 0 \\ -3 & 1 & 1 \end{pmatrix}.A \quad \underline{R_3 \to -\frac{1}{4}R_3}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 & 0 \\ 3 & -1 & 0 \\ -\frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} \end{pmatrix} \quad \underline{R_2 \to R_2 - 2R_3}$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 & 0 \\ \frac{3}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} \end{pmatrix}$$

$$\therefore A^{-1} = \begin{pmatrix} -2 & 1 & 0 \\ \frac{3}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{3}{4} & -\frac{1}{4} & -\frac{1}{4} \end{pmatrix}$$

(b)(i) $z = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$

$$\frac{\partial z}{\partial x} = x^2 \frac{1}{1+\left(\frac{y}{x}\right)^2} y.(-1)x^{-2} + \tan^{-1}\left(\frac{y}{x}\right).2x - y^2.\frac{1}{1+\left(\frac{x}{y}\right)^2}.\frac{1}{y}$$

$$= -y + 2x \tan^{-1}\frac{y}{x}$$

$$\frac{\partial^2 z}{\partial y \partial x} = -1 + 2x.\frac{1}{1+\left(\frac{y}{x}\right)^2}.\frac{1}{x}$$

$$= \frac{-x^2 - y^2 + 2x^2}{x^2 + y^2}$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{x^2 - y^2}{x^2 + y^2}$$

(ii) $z = \log_e\left(x^2 + y^2\right)$

$$\frac{\partial z}{\partial x} = \frac{1}{x^2 + y^2}.2x$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{2}{x^2 + y^2} + 2x\frac{(-1)(2x)}{\left(x^2 + y^2\right)^2} = \frac{2}{x^2 + y^2} - \frac{4x^2}{\left(x^2 + y^2\right)^2}$$

$$\frac{\partial z}{\partial y} = \frac{1}{x^2 + y^2}.2y$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{2}{x^2 + y^2} + \frac{2y.(-1)(2y)}{\left(x^2 + y^2\right)^2} = \frac{2}{x^2 + y^2} - \frac{4y^2}{\left(x^2 + y^2\right)^2}$$

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{4}{x^2 + y^2} - \frac{-4(x^2 + y^2)}{(x^2 + y^2)^2} = \frac{4}{x^2 + y^2} - \frac{4}{x^2 + y^2} = 0$$

$$\therefore \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$$

(iii) Associative property – multiplication

(PQ)R = P(QR)

P	Q	R	PQ	(PQ)R	QR	P(QR)
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	0	0	0
0	1	1	0	0	1	0
1	0	0	0	0	0	0
1	0	1	0	0	0	0
1	1	0	1	0	0	0
1	1	1	1	1	1	1