MPZ 3230 – Assignment #03
Academic year – 2006

1) (i) OA,OB,OC are three mutually perpendicular lines and H is a point on the plane ABC such that OH is perpendicular to the plane ABC. Prove that H is the orthocentre of the triangle ABC.

(ii) ABCD is a tetrahedron in which AB=AC=AD and BCD is an equilateral triangle. The angle between AB and AC is α and AB makes an angle β with the plane BCD. Prove that $4\cos\alpha + 3\cos 2\beta = 1$

2) (i) Prove that the straight lines joining the middle points of opposite edges of a tetrahedron ABCD are concurrent and bisect each other.

(ii) Prove also that if DA is perpendicular to BC and DB is perpendicular to CA then DC is perpendicular to AB and $DA^2 + BC^2 = DB^2 + CA^2 = DC^2 + AB^2$

3) (i) Find the root of $f(x) = \cos x - x$ by using Bisection Method. Repeat the procedure until 4 iteration.

(ii) Use Newton – Raphson method to find a real root of the equation $x^4 - x - 10$, correct to three decimal places, with 3 iteration.

(iii) solve the following system with Gaussian elimination method in matrix form

$$3x_1 - x_2 + 2x_3 = 12$$
$$x_1 + 2x_2 + 3x_3 = 11$$
$$4x_1 - 4x_2 - 2x_3 = 4$$

System has true solution $x_1=3, x_2=1$ and $x_3=2$.state the reasons for deviation of values in Gaussian elimination method and true solution.

4) The variables x_1, x_2, x_3, x_4 satisfy the equations,

$$-x_1 + x_2 - x_3 + 10x_4 = -1$$
$$2x_1 - x_2 + 4x_3 - 21x_4 = -3$$
$$-x_1 - x_2 - 6x_3 + 13x_4 = 11$$
$$-2x_1 + 3x_2 + x_3 + 19x_4 = -9$$

(i)Perform 3 steps of the Gauss – seidal iteration process on the above equations taking $(0,0,0,0)^T$ as the starting vector and working to 2 decimal place.

(ii)Solve the above system of equations using Jacobi method with **matrix form** taking initial guess $(0,0,0,0)^T$ and working to 2 decimal places with 3 iterations.

Please send your assignment **on or before 08.10.2006** to the following address.
Please send your answer with your address (write back of your answer sheet) and when you are doing assignment use both sides of paper.

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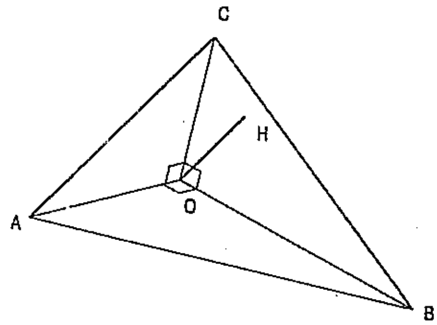
MPZ 3230 – Model answer – 03
Academic Year – 2006

(1)(i)

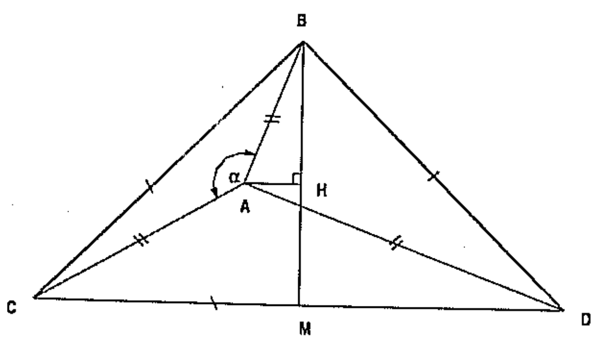


OC is perpendicular to OA and OB. (Given) Therefore OC is perpendicular to plane OAB and hence OC is perpendicular to AB.

Also OH is perpendicular to plane ABC. (Given) Therefore OH is perpendicular to AB. Above two results imply that AB is perpendicular to plane COH. Therefore AB is perpendicular to CH.

Similarly AC is perpendicular to BH. Therefore H is the orthocenter of triangle ABC.

(ii)



Let AB = AC = AD = a

Then BC = CD = BD = $2a\sin\frac{\alpha}{2}$

Note that angle $C\hat{A}D = \alpha =$ angle BAD
Since triangles BCA, CDA, DBA are congruent.
Let H be the foot of perpendicular from A upon plane BCD. Also let BH meet CD at M.
The AH is perpendicular plane BCD and triangles ABH, ACH, ADH are congruent.
Therefore BH = CH = DH and hence triangles BHC, BHD are congruent.

Consequently,

Angle CBM = angle DBM and triangles CBM and DBM are congruent.
Therefore BM is perpendicular to CD and M is the mid point of CD.
Also,

$$BH = \frac{2}{3}BM = \frac{2}{3}BC\cos\frac{\pi}{6} = \frac{2}{3}\left(2a\sin\frac{\alpha}{2}\right)\frac{\sqrt{3}}{2}$$

$$BH = BA\cos\beta = a\cos\beta$$

$$\Rightarrow a\cos\beta = \frac{2}{3}\left(2a\sin\frac{\alpha}{2}\right)\cdot\frac{\sqrt{3}}{2}$$

$$\Rightarrow \cos\beta = \frac{2}{\sqrt{3}}\sin\frac{\alpha}{2}$$

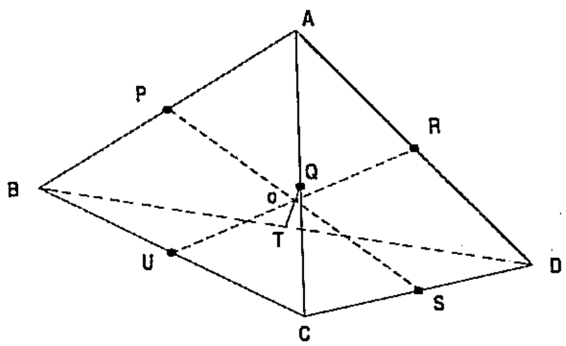
Therefore,

$$\cos 2\beta - 2\cos^2\beta - 1 = \frac{8}{3}\sin^2\frac{\alpha}{2} - 1 - \frac{4}{3}(1-\cos\alpha) - 1$$

$$\Rightarrow \frac{4}{3}\cos\alpha + \cos 2\beta = 1$$

$$4\cos\alpha + 3\cos 2\beta = 1$$

(2) (i)



P, Q, R, S, T, U are the mid points of AB, AC, AD, DC, DB, BC respectively.

PR // BD, US//BD, $\therefore$ PR//US

Similarly PU//RS.

$\therefore$ PUSR is a parallelogram.

Let mid point of PS be O.

Diagonals of the parallelogram PUSR, PS and UR bisect each other at O.

Similarly PS, QT bisect each other at O PS, UR, QT are concurrent at O and bisect each other.

(ii) $AC \perp BD$ is given

But PU // AC and US//BD

$\therefore PU \perp US$

Parallelogram PUSR is a rectangle,

$$OU = OS = OP = OR \quad \rightarrow (1)$$

Similarly parallelogram PQST is a rectangle,

$$\therefore OP = OQ = OT = OS \quad \rightarrow (2)$$

From (1) and (2) we have

$$OQ = OU = OT, \text{ and } O \text{ is on } QT.$$

(O is the mid point of QT)

$\therefore$ QT is a diameter of the circle passing through QT, U.

$\therefore$ QUT is a right angle.

$AD \perp BC$

$AC \perp BD$ ie. $QU \perp UT$

But QU//AB, UT//CD

$$\therefore AB \perp CD$$

$$DA^2 + BC^2 = 4PT^2 + 4TS^2$$
$$= 4PS^2 \quad (\therefore \text{PQST is a rectangle})$$

Similarly,

$$DB^2 + CA^2 = 4PS^2 \text{ and}$$
$$DC^2 + AB^2 = 4TQ^2$$

But $TQ = 2OT = 2OS = PS$

$$\therefore DC^2 + AB^2 = 4PS^2$$

$$\therefore DA^2 + BC^2 = DB^2 + CA^2 = DC^2 + AB^2$$

(3) (i) $f(x) = \cos x - x$

Let $x_0 = 0, \quad x_1 = 1$

$f(x_0) = \cos 0 - 0 = 1$; $\quad f(x_1) = \cos 1 - 1 = -0.4597$

$$\therefore f(x_0).f(x_1) < 0$$

Since f(x) is continuous from x_0 to x_1 and also as it changes sign, f(x) = 0 has a root between 0 and 1,

n	x_0	x_1	$\lvert x_1 - x_0 \rvert$	x_2	$f(x_2)$	$f(x_0).f(x_2)$	Range for x^*
1	0	1	1	0.5	0.3776	>0	x_2 to x_1
2	0.5	1	0.5	0.75	-0.0183	<0	x_0 to x_2
3	0.5	0.75	0.25	0.625	0.1860	>0	x_2 to x_1
4	0.625	0.75	0.125	0.6875	0.0853	>0	x_2 to x_1

Root of f(x) = 0.6875

(ii) $f(x) = x^4 - x - 10$

$f(1) = 1^4 - 1 - 10 = -10 < 0$

$f(2) = 2^4 - 2 - 10 = 4 > 0$

$\therefore$ According to the intermediate value theorem there exist a root between 1 & 2.But a root is closed to 2.

Therefore $x_0 = 2$.

By using Newton-Raphson method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad f'(x) = 4x^3 - 1$$

When $n = 0;\ x_0 = 2$

$$f(x_0) = f(2) = 4,\ f'(x_0) = 4.2^3 - 1 = 31$$

$$x_1 = 2 - \frac{4}{31} = 1.8710$$

$$|x_1 - x_0| = |1.8710 - 2| = 0.1290$$

When $n = 1;\ x_1 = 1.8710$

$$f(x_1) = (1.8710)^4 - 1.8710 - 10 = 0.3835$$

$$f'(x_1) = 4(1.8710)^3 - 1 = 25.1988$$

$$x_2 = 1.8710 - \frac{0.3835}{25.1988}$$

$$= 1.8558$$

$$|x_2 - x_1| = |1.8558 - 1.8710| = 0.0152$$

When $n = 2,\quad x_2 = 1.8558$

$$f(x_2) = (1.8558)^4 - (1.8558) - 10$$
$$= 0.0053$$

$$f'(x_2) = 4(1.8558)^3 - 1 = 24.5654$$

$$x_3 = 1.8558 - \frac{0.0053}{24.5654} = 1.8556$$

$$|x_3 - x_2| = |1.8556 - 1.8558| = 0.0002$$

Real root of f(x) = 1.8556

tween I

(iii) $3x_1 - x_2 + 2x_3 = 12$
$x_1 + 2x_2 + 3x_3 = 11$
$4x_1 - 4x_2 - 2x_3 = 4$

We can write above system as matrix from, $A\underline{x} = \underline{b}$

$$\begin{pmatrix} 3 & -1 & 2 \\ 1 & 2 & 3 \\ 4 & -4 & -2 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 12 \\ 11 \\ 4 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} 3 & -1 & 2 & 12 \\ 1 & 2 & 3 & 11 \\ 4 & -4 & -2 & 4 \end{array}\right) \xrightarrow[r_3 \to r_3 - \frac{4}{3}r_1]{r_2 \to r_2 - \frac{1}{3}r_1} \left(\begin{array}{ccc|c} 3 & -1 & 2 & 12 \\ 0 & 2.333 & 2.333 & 7 \\ 0 & -2.667 & -4.667 & -12 \end{array}\right)$$

$$\begin{pmatrix} 3 & -1 & 2 & 12 \\ 0 & 2.333 & 2.333 & 7 \\ 0 & 0 & -2 & -3.998 \end{pmatrix} r_3 \to r_3 - \left(\frac{-2.667}{2.333}\right) r_2$$

Backward Substituting,

$$x_3 = \frac{-3.998}{-2} = 1.999$$

$$x_2 = \frac{7 - (2.333)(1.999)}{2.333} = 1.001$$

$$x_1 = \frac{12 - (-1)(1.001) - 2(1.999)}{3} = 3.001$$

Solution: $x_1 = 3.001$
$x_2 = 1.001$
$x_3 = 1.999$

True solution: $x_1 = 3,\ x_2 = 1,\ x_3 = 2$

Reason: Due to rounding up in the row- column operations in Gaussian elimination method.

(4) $-x_1 + x_2 - x_3 + 10x_4 = -1$
$2x_1 - x_2 + 4x_3 - 21x_4 = -3$
$-x_1 - x_2 - 6x_3 + 13x_4 = 11$
$-2x_1 + 3x_2 + x_3 + 19x_4 = -9$

By Gauss-seidel method,

$$x_1^{(m+1)} = 1 + x_2^{(m)} - x_3^{(m)} + 10x_4^{(m)}$$

$$x_2^{(m+1)} = 2x_1^{(m+1)} + 4x_3^{(m)} - 21x_4^{(m)} + 3$$

$$x_3^{(m+1)} = \frac{1}{6}\left\{-x_1^{(m+1)} - x_2^{(m+1)} + 13x_4^{(m)} - 11\right\}$$

$$x_4^{(m+1)} = \frac{1}{19}\left\{-9 + 2x_1^{(m+1)} - 3x_2^{(m+1)} - x_3^{(m+1)}\right\}$$

n	x_1	x_2	x_3	x_4
0	0	0	0	0
1	1	5	$-\frac{17}{6} = -2.83$	$-\frac{115}{114} = -1.0088$
2	-1.26	10.349	-5.534	-1.949
3	-2.607	16.579	-8.385	-2.924

$x_1 = -2.61$
$x_2 = 16.58$
$x_3 = -8.38$
$x_4 = -2.92$

(ii) Jacobi Method,
We can write the system as matrix form

$$\underbrace{\begin{pmatrix} -1 & 1 & -1 & 10 \\ 2 & -1 & 4 & -21 \\ -1 & -1 & -6 & 13 \\ -2 & 3 & 1 & 19 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}}_{\underline{x}} = \underbrace{\begin{pmatrix} -1 \\ -3 \\ 11 \\ -9 \end{pmatrix}}_{\underline{b}}$$

Then $\underline{x}^{(k+1)} = D^{-1}\underline{b} - D^{-1}(L+u)\underline{x}^{(k)}$

$$D = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -6 & 0 \\ 0 & 0 & 0 & 19 \end{pmatrix}$$

$$L=\begin{pmatrix}0&0&0&0\\2&0&0&0\\-1&-1&0&0\\-2&3&1&0\end{pmatrix}$$

$$U=\begin{pmatrix}0&1&-1&10\\0&0&4&-21\\0&0&0&13\\0&0&0&0\end{pmatrix}$$

$$DD^{-1}=1$$

$$\left(\begin{array}{cccc|cccc}-1&0&0&0&1&0&0&0\\0&-1&0&0&0&1&0&0\\0&0&-6&0&0&0&1&0\\0&0&0&19&0&0&0&1\end{array}\right)\begin{array}{c}r_1\to\frac{r_1}{-1}\\r_2\to\frac{r_2}{-1}\\r_3\to\frac{r_3}{-6}\\r_4\to\frac{r_4}{19}\end{array}\left(\begin{array}{cccc|cccc}1&0&0&0&-1&0&0&0\\0&1&0&0&0&-1&0&0\\0&0&1&0&0&0&-\frac{1}{6}&0\\0&0&0&1&0&0&0&\frac{1}{19}\end{array}\right)$$

$$D^{-1}=\begin{pmatrix}-1&0&0&0\\0&-1&0&0\\0&0&-\frac{1}{6}&0\\0&0&0&\frac{1}{19}\end{pmatrix}$$

<u>**1st Iteration**</u>

$$x^{(1)}=\begin{pmatrix}-1&0&0&0\\0&-1&0&0\\0&0&-\frac{1}{6}&0\\0&0&0&\frac{1}{19}\end{pmatrix}\begin{pmatrix}-1\\-3\\11\\-9\end{pmatrix}-\begin{pmatrix}-1&0&0&0\\0&-1&0&0\\0&0&-\frac{1}{6}&0\\0&0&0&\frac{1}{19}\end{pmatrix}\begin{pmatrix}0&1&-1&10\\2&0&4&-21\\-1&-1&0&13\\-2&3&1&0\end{pmatrix}\begin{pmatrix}0\\0\\0\\0\end{pmatrix}$$

$$x^{(1)}=\begin{pmatrix}1\\3\\-\frac{11}{6}\\-\frac{9}{19}\end{pmatrix}$$

2nd iteration

$$x^{(2)}=\begin{pmatrix}-1&0&0&0\\0&-1&0&0\\0&0&-\frac{1}{6}&0\\0&0&0&\frac{1}{19}\end{pmatrix}\begin{pmatrix}-1\\-3\\11\\-9\end{pmatrix}-\begin{pmatrix}-1&0&0&0\\0&-1&0&0\\0&0&-\frac{1}{6}&0\\0&0&0&\frac{1}{19}\end{pmatrix}\begin{pmatrix}0&1&-1&10\\2&0&4&-21\\-1&-1&0&13\\-2&3&1&0\end{pmatrix}\begin{pmatrix}1\\3\\-\frac{11}{6}\\-\frac{9}{19}\end{pmatrix}$$

$$=\begin{pmatrix}1\\3\\-\frac{11}{6}\\-\frac{9}{19}\end{pmatrix}-\begin{pmatrix}-\frac{11}{114}\\-\frac{263}{57}\\\frac{193}{114}\\\frac{31}{114}\end{pmatrix}=\begin{pmatrix}\frac{125}{114}\\\frac{434}{57}\\\frac{-67}{19}\\\frac{-85}{114}\end{pmatrix}$$

3rd iteration

$$x^{(3)}=\begin{pmatrix}-1&0&0&0\\0&-1&0&0\\0&0&\frac{-1}{6}&0\\0&0&0&\frac{1}{19}\end{pmatrix}\begin{pmatrix}-1\\-3\\11\\-9\end{pmatrix}-\begin{pmatrix}-1&0&0&0\\0&-1&0&0\\0&0&\frac{-1}{6}&0\\0&0&0&\frac{1}{19}\end{pmatrix}\begin{pmatrix}0&1&-1&10\\2&0&4&-21\\-1&-1&0&13\\-2&3&1&0\end{pmatrix}\begin{pmatrix}\frac{125}{114}\\\frac{434}{57}\\\frac{-67}{19}\\\frac{-85}{114}\end{pmatrix}$$

$$=\begin{pmatrix}1\\3\\\frac{-11}{6}\\\frac{-9}{19}\end{pmatrix}-\begin{pmatrix}\frac{-70}{19}\\\frac{-427}{114}\\\frac{1049}{342}\\\frac{976}{1083}\end{pmatrix}=\begin{pmatrix}\frac{89}{19}\\\frac{769}{114}\\\frac{-838}{171}\\\frac{-1489}{1083}\end{pmatrix}=\begin{pmatrix}4.684\\6.746\\-4.901\\-1.372\end{pmatrix}$$

Solution:

$$\begin{pmatrix}x_1^{(3)}\\x_2^{(3)}\\x_3^{(3)}\\x_4^{(3)}\end{pmatrix}=\begin{pmatrix}4.68\\6.75\\-4.9\\-1.37\end{pmatrix}$$