THE OPEN UNIVERSITY OF SRI LANKA
DIPLOMA IN TECHNOLOGY– LEVEL 03
FINAL EXAMINATION - 2006
MPZ 3230 – ENGINEERING MATHEMATICS I
DURATION – THREE (03) HOURS

DATE : 21st April 2006 **TIME: 9.30 a.m. – 12.30 p.m.**

- **Answer only six (06) questions**
- **State any assumption you required.**
- **Do not spend more than 30 minutes for one problem.**
- **Show all your workings.**
- **All symbols are in standard notation.**

01. i. Define dot product and vector product of two vectors $\underline{a}$ and $\underline{b}$.

ii. A and B are two points and CD is a telephone line in space. AP and BQ are perpendiculars to line CD from A and B respectively.

A = (2, -2, 1) , B = (1, 7, 0)
C = (2, 0, -1) and line CD is parallel to the vector (1, 2, -1)

Using vectors, find

a) $\overrightarrow{AC}$ and $\overrightarrow{BC}$
b) Unit vector of line $\overrightarrow{CD}$
c) Perpendicular distances AP and BQ
d) Projection distances CP and CQ
e) Coordinates of P and Q
f) Angles $A\hat{P}B$ and $A\hat{Q}B$

02. i. Write vector expressions for

a) $\frac{d}{dt}(\underline{a}\,.\,\underline{b})$ and

b) $\frac{d}{dt}(\underline{a}\times\underline{b})$

c) $\frac{d}{dt}(\underline{a}\times\underline{b}\times\underline{c})$ where $\underline{a}$, $\underline{b}$ and $\underline{c}$ are vectors.

ii. The position vector of a particle varies with time according to the equation,

$$\underline{r}(t) = -(t^4 + 3t)\,\underline{i} + 8t^2\,\underline{j} + 4\,\underline{k}$$

Find,

a) The particle's velocity and acceleration vectors at t = 0.
b) The particle's velocity and acceleration vectors at t = 1.
c) The particles speed and its direction of motion at t = 1.

03. i. Define the determinant of a 3 x 3 matrix A, B are two square matrices and I is the identity matrix. Assuming the results $AA^{-1} = I$ and $|AB| = |A|\,|B|$, show that $|A^{-1}| = \frac{1}{|A|}$

ii. Show that $\begin{vmatrix} a-x & a-y & a-z \\ b-x & b-y & b-z \\ c-x & c-y & c-z \end{vmatrix} = 0$

04. i. Define the inverse of a matrix. State the conditions under which inverse of a matrix A exists.

ii. $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, Prove that

$A^{-1} = \frac{1}{k}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$, Where $k = (ad-bc)$. Hence deduce the inverse of the matrix $A = \begin{pmatrix} -3 & 4 \\ -6 & 7 \end{pmatrix}$

iii. The 3 currents in a electrical circuit are given by following equations.

$i_1 + 2i_2 + 3i_3 = 1$

$i_1 + 3i_2 + 5i_3 = 3$

$i_1 + 5i_2 + 12i_3 = -4$

a) Write these equations in matrix form.

b) **Using a matrix inversion method**, calculate the three currents of the electrical circuits.

05. Solve following differential equations.

a) $\dfrac{dy}{dx} + y\cot x = x^2$

b) $y'' - 4y' + 20y = 0$

c) $y'' - 4y' + 4y = 3e^{2x}$

06. i. Obtain the Laplace transform of

a) t^3 (b) $t\cos\omega t$, where ω is a constant.

ii. Find the inverse transform of $\dfrac{8s-2}{s^3+s^2-2s}$

iii. State convolution theorem for Laplace transform of $L[h(t)] = \dfrac{s}{(s-a)(s^2+\omega^2)}$.

Using above theorem find h(t) for a constant ω.

07. i. A storage tank is in the shape of a right circular cone, with its axis vertical and its vertex downwords. The volume of fluid in the tank is

$$V = \frac{1}{3}\pi r^2 h$$

Where h is the depth of fluid and r the radius of the fluid surface area.

Find an expression for dV/dt, the rate of change of volume with respect to time t. Further, assuming that h and r are not mutually independent and that h/r is constant, express dV/dt in terms of r and dr/dt.

Suppose that at some instant h = 5m, r = 3m and V is increasing at 500 m^3/s. What is the corresponding rate of change of depth?

ii. The transverse deflection y of a strut of length l under an axial compressive force P satisfies the equation.

$$EI\frac{d^2y}{dx^2} + Py = 0$$

Where E and I are constants.

Assuming the boundary conditions y(0) = y (l)=0, solve the equation and hence show that the smallest value of P which will cause a transverse deflection is $EI\,\pi^2/l^2$.

08. i. A, B and C are three events. Find an expression for P(A∪B∪C) in simple probabilities.

If A and B are mutually exclusive events, using a venn diagram, show that P(A)+P(B)+P(C)=P(A∪B∪C)+P(B∩C)+P(C∩A)

ii. C and D are two events. It is given that, P(C∪D)=0.45, P(C)=0.37, and P(D)=0.35. Calculate

a. P(C∩D) b. $P[(C \cap D)']$

c. P(C/D') and d. P(D'/C)

(Hint: For parts (c) and (d) use the following venn diagram result) [(C∩D)∪(C∩D')]=C

09. i. Steel girders are manufactured by three factories A, B and C. Each month factory A makes three times as many as factory B and factories B and C make the same number of girders. Of the girders made by factory A and of those made by C 2% are defective, of those made by B, 1.5% are defective. Two months production of girders of the 3 factories in put into a ware house. Given that one girder is chosen at random from the warehouse,

a. Find the probability that this item will not be defective.

b. Given that the girder is not defective, find the probability that it comes from factory B.

c. Given that the girder is defective, find the probability that it comes from factory B or C.

ii. A continuous random variable x has a probability density function of the form $f_x(x)=ce^{-3x}$ if $x \geq 0$
$= 0$ otherwise.

a. Find the value of C.

b. Find prob $(x \geq 2)$ and prob $(3 \leq x \leq 5)$

c. Find the value of d, for which prob $(x \leq d)=0.6$

d. Find mean and variance of x.

10. i. The following table given recordings of the temperature (θ) in a mechanical plant at a given time.

T	0	2	4	6	8	10
θ	75.00	74.25	72.13	60.54	53.21	43.86

a. At what time θ is equal to 68 degrees?

b. Calculate the time at which rate of decrease of temperature of the plant be zero for the cooling situation.

c. Compute $\frac{d^2\theta}{dt^2}$ for the time you have obtained in part (b).

ii. In computing the capacity of a reservoir, a contour interval of 1m was used. The areas measured over the contour intervals are given below.

Area	A1	A2	A3	A4	A5	A6
m^2	4156	5208	6303	7500	9200	10,501

Compute the volume capacity of the reservoir using Simpson's and Trapezoidal rules. Comment which method is more accurate.

11. The variables x_1, x_2, x_3, x_4 satisfy the equations,

$$\begin{aligned} -x_1 \quad +x_2 \quad -x_3 \quad +10x_4 &= -1 \\ 2x_1 \quad -x_2 \quad +4x_3 \quad -21x_4 &= -3 \\ -x_1 \quad -x_2 \quad -6x_3 \quad +13x_4 &= 11 \\ -2x_1 \quad +3x_2 \quad +x_3 \quad +19x_4 &= -9 \end{aligned}$$

and are such that $-3 \le x_n \le 3$ for n = 1, 2, 3, 4. Perform 3 steps of the gauss – seidal iteration process on the above equations, taking $(0, 0, 0, 0)^T$ as the starting vector and working to 1 decimal place. Comment briefly on the results obtained and the reason for these results.

12. **Answer any two parts out of (i), (ii) and (iii).**

i. P is any point outside the plane α and l is a line lying on α.

If O and A are the feet of perpendiculars from P to α and l respectively. Show that OA is perpendicular to l.

ii. a. State associative property based on truth table with respect to the multiplication property.

b. Prove De-Morgan's theorem using logical expressions.

iii. a. If $z=x^4+2x^2y+y^3$ and $x = \gamma \cos\theta$ and $y=\gamma \sin\theta$. Find $\frac{\partial z}{\partial \gamma}$ and $\frac{\partial z}{\partial \theta}$ in their simplest forms.

b. If $Z=x\ln(x^2+y^2) - 2y\tan^{-1}(\frac{y}{x})$ verify that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z + 2x$