

The Open University of Sri Lanka
 B.Sc/B.Ed Degree Programme
 Open Book Test (OBT) - 2010/2011
 Applied Mathematics – Level 5
 AMU3186/AME5186 – Quantum Mechanics



Duration :- One and half hours

Date:- 18.03.2011

Time:- 4.00p.m.-5.30p.m.

Answer ALL Questions

1. An X ray photon of wave length $\lambda = 10^{-10} m$ is incident of a stationary electron, where $\lambda_c = \frac{h}{mc}$ is the Compton wave length.

(i) Show that for Compton scattering,

$\Delta\lambda = \lambda' - \lambda = 2\lambda_c \sin^2 \frac{\theta}{2}$, where m is the mass of the electron, λ is the wave length of the incident X ray and λ' is the wave length of X ray scattered through an angle θ .

(ii) Calculate the Compton shift.

(iii) Calculate the kinetic energy of the recoiling electron.

(iv) Show that the maximum kinetic energy of the recoiling electron is given by

$$K.E_{\max} = \frac{hc}{\lambda} \left(\frac{2\lambda_c}{\lambda + 2\lambda_c} \right).$$

$$m_e = 9.108 \times 10^{-31} \text{ Kg}, \quad c = 3 \times 10^8 \text{ ms}^{-1}, \quad h = 6.625 \times 10^{-34} \text{ Js}$$

2. Consider a particle with normalized wave function

$$\psi(x) = A \sin\left(\frac{3\pi x}{2a}\right) \quad ; 0 \leq x \leq a \quad \text{where } a \text{ is a constant.}$$

(a) Determine the normalized constant, A.

(b) Calculate the mean values of x, x^2 and \hat{p}_x with respect to ψ .

3. (a) Define the commutator of two operators \hat{A} and \hat{B} .

(b) Evaluate the following commutators.

(i) $[\hat{x}, \hat{p}_x]$

(ii) $[\hat{H}, \hat{x}]$

(iii) $[\hat{T}, \hat{p}_x]$

Where \hat{H} is one dimensional Hamiltonian operator, \hat{p}_x is momentum operator and \hat{T} is kinetic energy operator.

(c) If \hat{A} and \hat{B} are Hermitian operators, show that $i(\hat{A}\hat{B}-\hat{B}\hat{A})$ is hermitian.

(d) Operator $\hat{\Pi}$ is defined as $\hat{\Pi}f(x) = f(-x)$. Prove that $\hat{\Pi}$ is Hermitian and find eigen values.