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The Open University of Sri Lanka
B.Sc/B.Ed Degree Programme
Closed Book Test (CBT) - 2010/2011
Applied Mathematics – Level 5
AMU3186/AME5186 – Quantum Mechanics
Duration :- One and half hours

Date:- 04-05-2011 **Time:- 4.00 p.m. – 5.30 p.m.**

Answer All Questions.

01. If $\hat{A}$ is an operator corresponding to a quantum observable and $\langle \hat{A} \rangle$ is the corresponding expectation value, show that $\frac{d\langle \hat{A} \rangle}{dt} = \frac{1}{i\hbar}\left\langle \left[\hat{A}, \hat{H}\right] \right\rangle + \left\langle \frac{\partial \hat{A}}{\partial t} \right\rangle$, where $\hat{H}$ is the time independent Hamiltonian operator for the quantum system.
Hence show that $\frac{d\langle \hat{x}^2 \rangle}{dt} = \frac{1}{m}\left[\langle \hat{x}\hat{P}_x \rangle + \langle \hat{P}_x \hat{x} \rangle\right]$, where $\hat{x}$ and $\hat{P}_x$ are position operator and momentum operator respectively.

02. In the usual notation consider a particle described by a Gaussian wave packet.

$$\psi(x) = A\exp\left[\frac{-(x-x_0)^2}{8a}\right]; \quad \text{where } x_0 \text{ and } a \text{ are constants.}$$

a) If ψ is normalized calculate A.
b) Calculate $\langle \Delta x \rangle$.

Use $\int_{-\infty}^{+\infty} e^{-\alpha^2 y^2} dy = \frac{\sqrt{\pi}}{\alpha}$; where α is a constant.

03. A particle of mass m and energy E moves in the positive x direction. Square well potential is defined by ,

$$V(x) = \begin{cases} 0 \;; x \le 0 \\ -V_0 \;; 0 \le x \le a \\ 0 \;; a < x \end{cases}$$

(i) Find the wave function $\psi(x)$ for each regions given above for case, $E > 0$ and $-V_0 < E < 0$.

(ii) Write down the equations to calculate transmission coefficient for the case $E > 0$ by applying boundary conditions.