The Open University of Sri Lanka
B.Sc/B.Ed Degree Programme
Open Book Test (OBT) - 2010/2011
Applied Mathematics – Level 4
AMU 2184/AME 4184 – Newtonian Mechanics
Duration :- One and half hours

Date:- 15.03.2011 **Time:- 4.00p.m.-5.30p.m.**

Answer **ALL** Questions

1. A particle of mass $3m$ is attached to the mid point of light elastic string of modulus mg whose ends are attached to two fixed points with distance $7l$ apart in a vertical line. The natural length of the string is $2l$. Find the equilibrium position of the particle.
   The particle is slightly disturbed from rest in a vertical direction. Show that the subsequent motion of the particle is simple harmonic of period $\pi\sqrt{\frac{2l}{g}}$.
   Find the maximum speed of the particle during this motion.

2. A particle is projected vertically upwards from O with speed u in a medium, which offers a resistance $\frac{kv^2}{a+y}$ per unit mass, where v is the speed of the particle, y is the height above the fixed point O and a and $k\left(\neq -\frac{1}{2}\right)$ are constants.
   Show that
   $(a+h)^{2k+1} = a^{2k+1}\left\{1+\frac{u^2(2k+1)}{2ag}\right\}$, where h is the maximum height attained by the particle.
   Considering downwards motion of the particle, write down equations to find its velocity at time t.

3. (i) The acceleration of an object is given by $\ddot{\underline{r}}(t) = 2\underline{i} - 6te^{-3t}\underline{j}$.
   Given that at time $t=0$, $\underline{v}(0) = \frac{2}{3}\underline{i}$ and $\underline{r}(0) = \frac{1}{3}\underline{j}$.
   Find the position vector at any time t.

(ii) A golf ball is driven from the tee O with speed u_0 at an angle α to the horizontal. Throughout its motion the only force acting on the particle is the force due to gravity.

a) Find the position vector $\underset{\sim}{r}(t)$ at time t, of the golf ball with O as the origin.

b) Show that horizontal range R of the particle is given by $R = \dfrac{2u_0^2 \tan\alpha}{g\left(1+\tan^2\alpha\right)}$.

The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme – Level 04
Closed Book Test (CBT) - 2010/2011
Applied Mathematics
AMU2184/AME4184 –Newtonian Mechanics

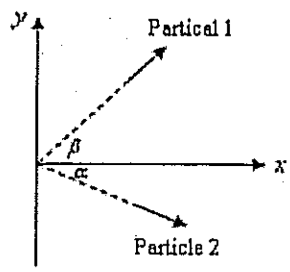
Duration: - One and Half Hours

Date:- 08-04-2011 **Time:- 4.00 p.m. – 5.30 p.m.**

Answer All Questions.

01. (i) A system consists of n particles and if $\underline{F}_i^{ext}$ is the external force acting on the i^{th} particle of the mass m_i and $\underline{F}_{ij}$ is the internal force acting on the i^{th} particle due to j^{th} particle of mass m_j. Show that the equation of motion for the system is given by $\underline{F}^{ext} = M\ddot{\underline{r}}$ where $\underline{F}^{ext}$ is the resultant external force acting on the system, M is the total mass of the system and $\underline{r}$ is the position vector of the centre of mass.

(ii) Two particles collide. Before the interaction, particle 1 of mass 2kg has velocity $0.5\,\underline{i}\ ms^{-1}$ and particle 2 of mass 3kg is stationary. After the interaction particle 1 has speed $3\,ms^{-1}$ and particle 2 has speed $2\,ms^{-1}$ and moves in the xy plane in the directions shown in the diagram below. Find the angles α and β.



02. (i) Establish the formula $\underline{F}(t) = m(t)\frac{d\underline{v}}{dt} + \frac{dm}{dt}\underline{u}$ for the motion of a particle of varying mass $m(t)$ moving with velocity $\underline{v}$ under a force $\underline{F}(t)$, matter being emitted at a rate $\frac{dm}{dt}$ with velocity $\underline{u}$ relative to the particle.

(ii) A rocket of initial total mass m propels itself by ejecting mass at a constant rate k per unit time with speed u relative to the rocket. At a time the rocket is moving with speed v vertically upwards near the Earth's surface against constant gravity. Show that $(1-kt)\frac{dv}{dt} = ku - g(1-kt)$. Given that $v = 0$ when $t = 0$, Find the speed v at time t.

03. (i) With usual notation show that the velocity and acceleration components in plane polar coordinates are given by $\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta$ and $\underline{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\underline{e}_r + \frac{1}{r}\frac{d}{dt}\left(r^2\dot{\theta}\right)\underline{e}_\theta$.

(ii) A particle (A) of mass m, kept on a horizontal table is connected to a light inextensible string passing through a small hole in the table, with another particle (B) of mass $2m$ hanging from the other end. If A is projected horizontally with velocity $3\sqrt{ga}$ perpendicular to the string with OA$=a$, show that in the subsequent motion, the length $r=$ OA satisfies the differential equation, $3\frac{d^2r}{dt^2}-\frac{9a^3g}{r^3}+2g=0$.