The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme
Final Examination-2010/2011
AMU 3187/ AME 5187- Mathematical Methods II
APPLIED MATHEMATICS-LEVEL 05

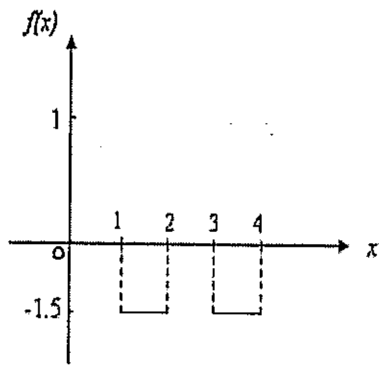
Duration: Two Hours.

Date:12.01.2011	**Time: 1.00 p.m.- 3.00p.m.**

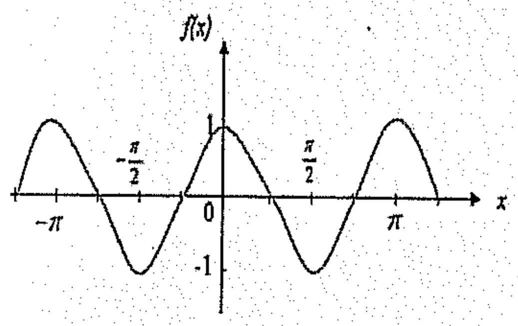
Answer FOUR questions only.

1. (i) Define analytically the periodic functions shown below:

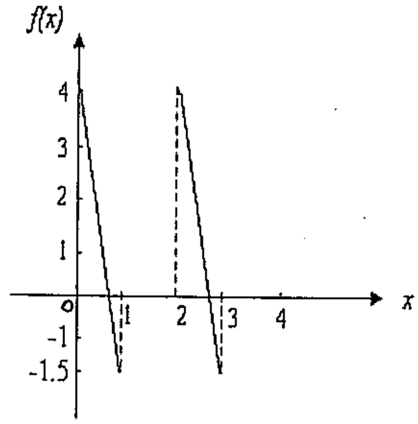
(a)



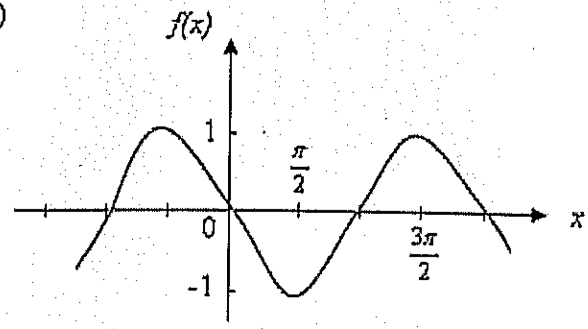
(b)



(c)



(d)



(ii) Find the period of each function given in part (i).

(iii) Assume that the Fourier coefficients of each function in part (i) are given. Explain how to obtain the Fourier coefficients of the following periodic functions by using them.

(a) $f(x) = 4\cos 2x$

(b) $f(x) = \begin{cases} \frac{1}{2}(8-11x) & 0 < x \le 1 \\ \frac{1}{2} & 1 < x < 2 \end{cases}$

2. (i) Sketch at least two periods for the graphs of the functions defined by

(a) $f(x) = \begin{cases} \sin x, & 0 \le x \le \pi \\ 0, & \pi \le x < 2\pi \end{cases}, \quad f(x+2\pi) = f(x)$

(b) $f(x) = \sin x \quad 0 \le x < \pi, \quad f(x+\pi) = f(x)$

(c) $f(x) = \frac{1}{p}x, \quad 0 \le x < p, \quad f(x+p) = f(x)$

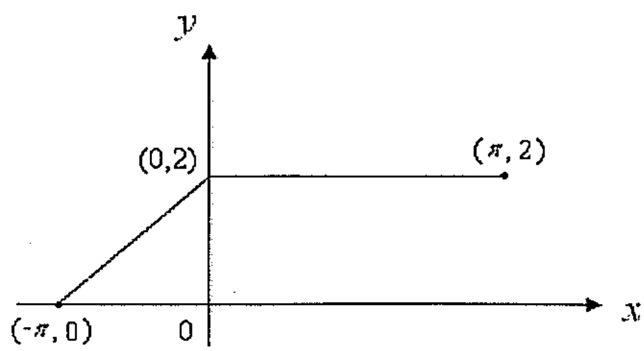
(d) $f(x) = \begin{cases} x, & 0 \le x \le 1, \\ 2-x, & 1 \le x < 2, \end{cases} \quad f(x+2) = f(x)$

(ii) (a) Sketch the odd and even extensions on the interval [-4, 4] for the function

$$f(x) = \begin{cases} x^2 & 0 < x \le 1 \\ -2x+3 & 1 < x \le 3. \\ 3x-12 & 3 < x < 4 \end{cases}$$

(b) Find its Fourier half-range sine series.

3. (i) Find the Fourier series corresponding to the function $f(x)$ such that the graph of $y = f(x)$ consists of the two line segments shown in the following figure.



(ii) Identify each of the following functions as being even or odd:

(a) $f(x) = x^n$, n is even.

(b) $f(x) = x^n$, n is odd.

(c) $f(x) = |x|$

(d) $f(x) = \sin x^2$.

4. Given the boundary value problem

$$\frac{d^2y}{dx^2} + \mu y = 0$$

$$y(-p) = y(p) \text{ and } y'(-p) = y'(p),$$

(i) Show that it is a Sturm-Liouville problem.

(ii) Find the eigenvalues and eigenfunctions.

(iii) Verify that the eigenfunctions are mutually orthogonal in the interval $-p \le x \le p$.

5. The Legendre polynomial of degree n, $P_n(x)$, is given by the expansion $(1-2xt+t^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)t^n$ for sufficiently small $|t|$.

Using this expansion prove that

(i) $P_n(1) = 1$.

(ii) $P_n(-1) = (-1)^n$.

(iii) $(n+1)P_{n+1}(x) + nP_{n-1}(x) = (2n+1)xP_n(x), \quad n = 1, 2, 3, \ldots$

(iv) Using part (iii) or otherwise show that $\|P_n(x)\| = \sqrt{\dfrac{2}{2n+1}}$.

6. Solve the following boundary value problem with mixed boundary conditions.

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0, \qquad 0<x<a, \quad 0<y<b$$

$$\frac{\partial u}{\partial y}(x,0)=0, \qquad 0<x<a$$

$$u(x,b)=0, \qquad 0<x<a$$

$$u(0,y)=1, \qquad 0<y<b$$

$$u(a,y)=0, \qquad 0<y<b.$$