The Open University of Sri Lanka
B.Sc. / B.Ed. Degree Programme – Level 05
Final Examination – 2010/2011
Applied Mathematics
AMU 3184/AME 5184 – Dynamics

Duration :- Two Hours

Date :- 08.01.2011 Time:- 1.00 p.m. – 3.00 p.m.

Answer Four Questions Only.

1. (a) Obtain the components of the velocity and acceleration in intrinsic coordinates, of a particle moving along a curve in a plane.

   (b) A particle P moves on the outside of an ellipse of eccentricity e whose major axis is vertical. P starts from rest at the highest point. Show that P will leave the curve at a point where the eccentric angle ϕ is given by $e^2\cos^3\phi = 3\cos\phi - 2$.

2. (a) Show that in spherical polar coordinates, the components of the velocity and acceleration are given by $\dot{\underline{r}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\sin\theta\dot{\phi}\underline{k}$ and

$$\ddot{\underline{r}} = \left(\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta\right)\hat{r} + \left(\frac{1}{r}\frac{d}{dt}\left(r^2\dot{\theta}\right) - r\sin\theta\cos\theta\dot{\phi}\right) + \frac{1}{r\sin\theta}\frac{d}{dt}\left(r^2\sin^2\theta\dot{\phi}\right)\hat{\phi}$$

   respectively.

   (b) A particle moves on a smooth sphere of radius a under no forces except the reaction of the surface. If the particle is released from rest at a point with spherical polar coordinates $\theta = \beta,\ \phi = 0$. Show that its path is given by the equation $\cot\theta = \cot\beta\cos\phi$.

3. (a) Obtain, in the usual notation, the equation $\frac{\partial^2 r}{\partial t^2} + 2\underline{\omega} \times \frac{\partial \underline{r}}{\partial t} = -g\underline{k}$ for the motion of a particle relative to the rotating earth.

   (b) A projectile located at a point of latitude λ is projected with speed v_0 in a Northward direction at an angle α to the horizontal. Find the position of the projectile after time t.

4. (a) With the usual notation, show that the Lagrange's equations of motion for a holonomic system are given by $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right)-\frac{\partial T}{\partial q_j}=Q_j,\ \ j=1,2\ldots,n.$

(b) A light string passes over a fixed smooth pully. It carries a mass of $8m$ at one end, the other end being attached to a smooth pulley of mass $4m$ over which passes a light string whose ends carry masses $3m$ and m.

(*i*) Assuming that the system starts from rest and neglecting moments of inertia of the pulleys set up the Lagrangian of the system.

(*ii*) Find the accelerations of the moveable pulley and the masses.

5. (*a*) Derive Euler's equations of motion of a rigid body rotating about a fixed point.

(*b*) If a body moves under no forces about a point O and if H is the angular momentum about O and T the kinetic energy of the body then show that H and T are conserved.

(c) A rigid body moves about a point O under no forces. The principal moments of inertia of a body at O being $3A$, $5A$, $6A$. The body is initially rotated with an angular velocity $\underline{\omega}=(\omega_1,\ \omega_2,\ \omega_3)$ about the principal axes where $\omega_1=n$, $\omega_2=0$ and $\omega_3=n$. Show that any time t, $\omega_2=\frac{3n}{\sqrt{5}}\tanh\left(\frac{nt}{\sqrt{5}}\right)$.

6. (a) Define the Hamiltonian H of a holonomic system and derive in the usual notation, Hamilton's equations of motion, $\frac{\partial H}{\partial p_i}=\dot{q}_i,\ \ \frac{\partial H}{\partial q_i}=-\dot{p}_i$.

(b) Show that if the position vectors $\underline{r}_i=\underline{r}_i(q_1,\ q_2\cdots q_n)$, $i=1,2,\ldots N$ of the particles in terms of the generalised coordinates q_j $(j=1,2,\ldots n)$ do not involve t explicitly then $H=T+V$.

(c) Using cylindrical polar coordinates $(\rho,\ \theta,\ z)$ write the Hamilton's equations of motion for a particle of mass m moving in a force field of potential $V(\rho,\ \theta,\ z)$.