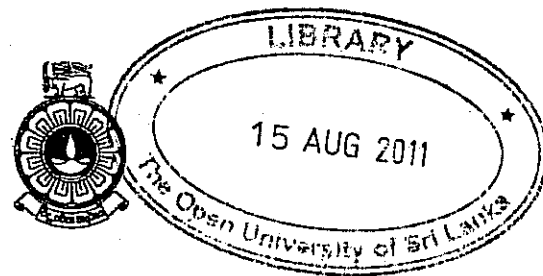


The Open University of Sri Lanka  
 B.Sc. / B.Ed. Degree Programme – Level 05  
 Final Examination – 2010/2011  
 Applied Mathematics  
 AMU 3184/AME 5184 – Dynamics



Duration :- Two Hours

Date :- 08.01.2011

Time:- 1.00 p.m. – 3.00 p.m.

**Answer Four Questions Only.**

1. (a) Obtain the components of the velocity and acceleration in intrinsic coordinates, of a particle moving along a curve in a plane.  
 (b) A particle  $P$  moves on the outside of an ellipse of eccentricity  $e$  whose major axis is vertical.  $P$  starts from rest at the highest point. Show that  $P$  will leave the curve at a point where the eccentric angle  $\phi$  is given by  $e^2 \cos^3 \phi = 3 \cos \phi - 2$ .
  
2. (a) Show that in spherical polar coordinates, the components of the velocity and acceleration are given by  $\dot{\underline{r}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\sin\theta\dot{\phi}\hat{\phi}$  and  

$$\ddot{\underline{r}} = (\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2 \sin^2 \theta)\hat{r} + \left( \frac{1}{r} \frac{d}{dt}(r^2\dot{\theta}) - r\sin\theta\cos\theta\dot{\phi} \right) \hat{\theta} + \frac{1}{r\sin\theta} \frac{d}{dt}(r^2 \sin^2 \theta \dot{\phi}) \hat{\phi}$$
 respectively.  
 (b) A particle moves on a smooth sphere of radius  $a$  under no forces except the reaction of the surface. If the particle is released from rest at a point with spherical polar coordinates  $\theta = \beta$ ,  $\phi = 0$ . Show that its path is given by the equation  $\cot \theta = \cot \beta \cos \phi$ .
  
3. (a) Obtain, in the usual notation, the equation  $\frac{\partial^2 r}{\partial t^2} + 2\omega \times \frac{\partial r}{\partial t} = -g\hat{k}$  for the motion of a particle relative to the rotating earth.  
 (b) A projectile located at a point of latitude  $\lambda$  is projected with speed  $v_0$  in a Northward direction at an angle  $\alpha$  to the horizontal. Find the position of the projectile after time  $t$ .

4. (a) With the usual notation, show that the Lagrange's equations of motion for a holonomic system are given by  $\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_j} \right) - \frac{\partial T}{\partial q_j} = Q_j$ ,  $j = 1, 2, \dots, n$ .
- (b) A light string passes over a fixed smooth pulley. It carries a mass of  $8m$  at one end, the other end being attached to a smooth pulley of mass  $4m$  over which passes a light string whose ends carry masses  $3m$  and  $m$ .
- (i) Assuming that the system starts from rest and neglecting moments of inertia of the pulleys set up the Lagrangian of the system.
- (ii) Find the accelerations of the moveable pulley and the masses.
5. (a) Derive Euler's equations of motion of a rigid body rotating about a fixed point.
- (b) If a body moves under no forces about a point  $O$  and if  $H$  is the angular momentum about  $O$  and  $T$  the kinetic energy of the body then show that  $H$  and  $T$  are conserved.
- (c) A rigid body moves about a point  $O$  under no forces. The principal moments of inertia of a body at  $O$  being  $3A$ ,  $5A$ ,  $6A$ . The body is initially rotated with an angular velocity  $\underline{\omega} = (\omega_1, \omega_2, \omega_3)$  about the principal axes where  $\omega_1 = n$ ,  $\omega_2 = 0$  and  $\omega_3 = n$ . Show that any time  $t$ ,  $\omega_2 = \frac{3n}{\sqrt{5}} \tanh \left( \frac{nt}{\sqrt{5}} \right)$ .
6. (a) Define the Hamiltonian  $H$  of a holonomic system and derive in the usual notation, Hamilton's equations of motion,  $\frac{\partial H}{\partial p_i} = \dot{q}_i$ ,  $\frac{\partial H}{\partial q_i} = -\dot{p}_i$ .
- (b) Show that if the position vectors  $\underline{r}_i = \underline{r}_i(q_1, q_2, \dots, q_n)$ ,  $i = 1, 2, \dots, N$  of the particles in terms of the generalised coordinates  $q_j$  ( $j = 1, 2, \dots, n$ ) do not involve  $t$  explicitly then  $H = T + V$ .
- (c) Using cylindrical polar coordinates  $(\rho, \theta, z)$  write the Hamilton's equations of motion for a particle of mass  $m$  moving in a force field of potential  $V(\rho, \theta, z)$ .