The Open University of Sri Lanka
B.Sc/B.Ed Degree Programme
AMU 3182/AME 5182 –Mathematical Methods I
Level 05-Applied Mathematics
FINAL EXAMINATION -2010/2011

Duration :- Two hours.

Date :- 03-01-2011 **Time:- 9.30 a.m.-11.30 a.m.**

Answer FOUR Questions Only.

01. (a) Find the general solution of the simultaneous differential equations:

$$\frac{d\underline{X}}{dt}=\begin{pmatrix}1 & 0 & 0\\ 2 & 1 & -2\\ 3 & 2 & 1\end{pmatrix}\underline{X},\ \text{where } \underline{X}=\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}.$$

(b) Using (a) or otherwise, write down the general solution of the system:

$$\frac{d^2\underline{X}}{dt^2}=\begin{pmatrix}1 & 0 & 0\\ 2 & 1 & -2\\ 3 & 2 & 1\end{pmatrix}\underline{X},\ \text{where } \underline{X}=\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}.$$

02. (a) Find the general solution of the simultaneous differential equations:

$$\frac{dx}{dt}=x+2y+e^t,$$

$$\frac{dy}{dt}=-x+4y+2.$$

(b) Solve the differential equation:

$$\frac{d^2y}{dx^2}+4\frac{dy}{dx}+3y=8\cos x-6\sin x.$$

Find also the particular solution for which $y=4$ and $\frac{dy}{dx}=-2$ when $x=0$.

(c) Solve the following homogeneous linear differential equation using a suitable substitution:

$$9x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+4y=0.$$

03. (a) Find the general solution of each of the following equations.

(*i*) $\dfrac{\partial^2 u}{\partial y^2}-\dfrac{1}{y}\dfrac{\partial u}{\partial y}=0.$ $\quad(y\neq 0)$

(*ii*) $\dfrac{\partial^2 u}{\partial x\partial t}-\dfrac{1}{t}\dfrac{\partial u}{\partial x}=x.$ $\quad(t\neq 0),$ where $u=u(x,t)$.

(b) Hence or otherwise, use the change of variables
$\zeta=x^2+y^2,\quad \phi=y,$
to find the general solution of the equation

$$y^2\frac{\partial^2 u}{\partial x^2}-2xy\frac{\partial^2 u}{\partial x\partial y}+x^2\frac{\partial^2 u}{\partial y^2}-\frac{y^2}{x}\frac{\partial u}{\partial x}-\frac{x^2}{y}\frac{\partial u}{\partial y}=0 \qquad (x\neq 0,\ y\neq 0).$$

04. (a) Find the surface passing through the parabolas $u=0,\ y^2=4ax$ and $u=1,\ y^2=-4ax$ and satisfying the equation $x\dfrac{\partial^2 u}{\partial x^2}+2\dfrac{\partial u}{\partial x}=0.$

(b) Show that surface of revolution satisfying the differential equation $\dfrac{\partial^2 u}{\partial x^2}=12x^2+y^2$ and touching the plane $u=0$ is $u=\left(x^2+y^2\right)^2$.

05. (a) Find the general solution for each of the following partial differential equations by using the integrating factor method:

(*i*) $y\dfrac{\partial^2 u}{\partial y\partial x}-\dfrac{\partial u}{\partial x}=xy^2\cos(xy)$

(*ii*) $\dfrac{\partial^2 u}{\partial y^2}-x\dfrac{\partial u}{\partial y}=x^2$

(*iii*) $2y\dfrac{\partial u}{\partial y}+y^2\dfrac{\partial^2 u}{\partial y^2}=0.$

(b) Find the general solution of the equation $\dfrac{\partial^2 u}{\partial x^2}-4\dfrac{\partial u}{\partial t}=0.$
Find the particular solution which satisfies the conditions
$u(x,0)=\sin x$, and
$\dfrac{\partial u}{\partial t}(x,0)=0.$

06. Solve the partial differential equation:
$\dfrac{\partial^2 u}{\partial t^2}=c^2\dfrac{\partial^2 u}{\partial x^2},\ 0<x<a,\ t>0$ given that
$u(x,0)=b\sin\left(\dfrac{\pi x}{a}\right),\quad \dfrac{\partial u}{\partial t}(x,0)=0,$ and $u(0,t)=0,\quad u(a,t)=0.$