The Open University of Sri Lanka B.Sc. / B.Ed. Degree Programme Closed Book Test (CBT) - 2010/2011 Applied Mathematics — Level 04 APU2142/APE4142 —Newtonian Mechanics I



Duration :- One and Half Hours

Date: - 08-04-2011

Time: - 4.00 a.m. - 5.30 p.m.

Answer All Questions.

- 01. (i) Establish the formula $\underline{F}(t) = m(t) \frac{d\underline{v}}{dt} \frac{dm}{dt} \underline{u}$ for the motion of a particle of varying mass m(t) moving with velocity \underline{v} under a force $\underline{F}(t)$, matter being condensed at a rate $\frac{dm}{dt}$ with velocity \underline{u} relative to the particle.
 - (ii) A raindrop falls vertically from rest through a mist under the influence of gravity. It grows by condensation of mist droplets, initially at rest, on its surface. After falling for a time t, the mass of the raindrop is Me^{kt} , where M and k are positive constants. After a time T the mass has increased to 2M. Determine the value of k in terms of T.

Also, formulate the equation of motion of the raindrop and use the equation to:

- (i) find the speed of the raindrop at time T.
- (ii) determine how far it has fallen in time T in terms of g and T.
- 02. The kinetic energy of a rigid body which is rotating about a fixed point O with angular velocity ω is given by $\frac{1}{2}I_0\omega^2$, where I_0 is the moment of inertia about axis of rotation.

A uniform rod AB of mass m and length 4l is free to rotate in a vertical plane about a smooth horizontal axis through its mid-point. Particles of mass 3m and m are attached to the ends A and B respectively.

- (a) The rod is held with AB horizontal and then released from rest. Find, in terms of l and g, the angular speed of the rod when AB is vertical.
- (b) If instead, the rod is initially vertical with A below B and is then given an angular speed of $\sqrt{g/2l}$, calculate the angle between AB and the downward vertical when the rod first comes to rest.

03. Rate of change of angular momentum of a rigid body is given by $\underline{H} = \underline{M}$,

where
$$\underline{M} = \sum_{i=1}^{N} \underline{r}_{i} \times \underline{F}_{i}$$
.

A uniform circular disc, centre C, of mass m and radius r can rotate in a vertical plane about a smooth horizontal axis perpendicular to its plane through a point A on its rim. Initially, the disc is held at rest with AC horizontal. It is then released. Find the components of the force on the axis when AC-makes an angle θ with the downward vertical.

Calculate the magnitude of the force on the axis

- (a) when AC is vertical,
- (b) when $\theta = \frac{\pi}{2}$ and when it is greatest.