The Open University of Sri Lanka
B.Sc/ B.Ed Degree Programme – Level 05
Final Examination - 2011/2012
Applied Mathematics
APU3244–Graph Theory

Duration: - Three Hours

Date: - 06-12-2012. **Time:- 1.30 p.m. – 4.30 p.m.**

Answer Five Questions only.

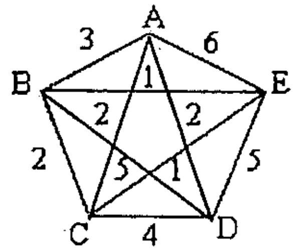
01. (a) Draw each of the following graphs if they exist, if not explain why such graphs do not exist:
    - (i) A bipartite graph that is regular of degree 2,
    - (ii) A cube graph with 5 vertices,
    - (iii) A complete graph that is a wheel.

(b) Prove that if a graph G is bipartite then each cycle of G has an even length.

(c) Three unfriendly neighbors use the same water, oil and treacle utilities. In order to avoid meeting, they wish to build non-crossing paths from each of their houses to each of the three utilities. Can this be done? Justify your answer.

02. (a) Define a Hamiltonian circuit of a graph G.
    - (i) For which values of n and m does the complete bipartite graph $K_{m,n}$ have a Hamiltonian circuit,
    - (i) Give an example that a connected graph need not have Hamiltonian circuit.

(b) Traveling salesman wishes to visit five cities A, B, C, D, E and return to his starting point, covering the least possible total distance. Find the minimum total distance he travels?



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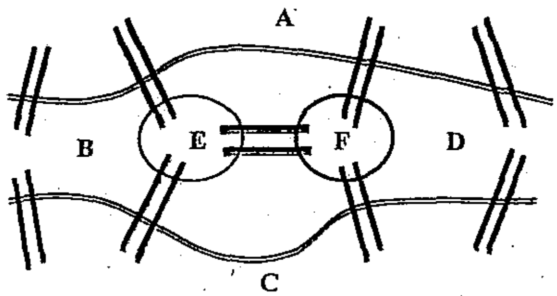
03. (a) Define an Eulerian trial and give an example to illustrate your definition.
Prove that if a connected graph G is Eulerian then the degree of each vertex of G is even.

(b) Show that one can cross all the bridges shown in the following map exactly once and return to the starting point.



Here distances are in kilometers such that AB=1, BC= 2, CD=1, DA=1, AF=5, FC=1, CE=4, EA=1, EF=6.

Suppose a postman wishes to deliver the letters by covering least possible total distance, then

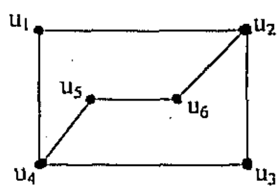
(i) find the shortest distance from E to F,

(ii) find the total distance of the Eulerian trial.

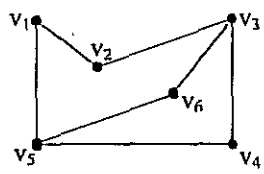
Hence determine the least possible total distance he has to travel in order to finish his job and return to the starting point.

04. (a) Let $G_1 = (V(G_1), E(G_1))$ and $G_2 = (V(G_2), E(G_2))$ be two graphs.
Define an isomorphism $f: G_1 \to G_2$.

(b) Show that the following graphs G_1 and G_2 are isomorphic:



G_1



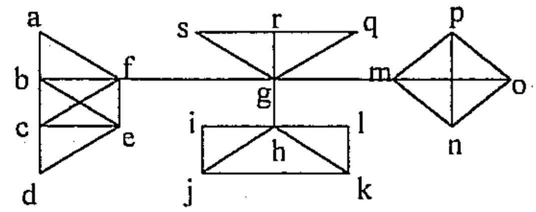
G_2

Write down the adjacency matrices A_{G_1} and A_{G_2} of the graphs G_1 and G_2 respectively.
Hence, determine whether the isomorphism f preserves edges.
Are $\overline{G_1}$ and $\overline{G_2}$ isomorphic? Justify your answer.

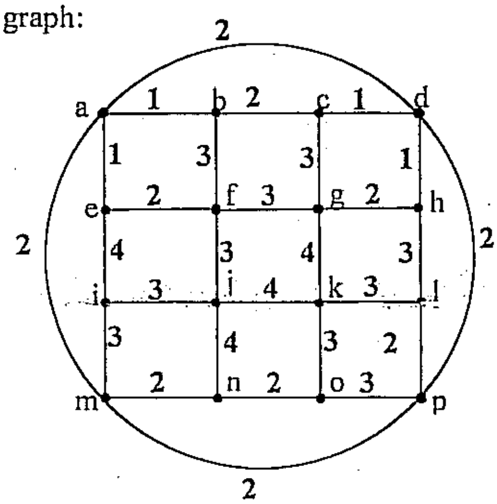
05. (a) Let T be a graph with n vertices. Prove that if T is a tree then T contains no cycle and has $n-1$ edges.

Draw all non-isomorphic spanning trees of $K_{3,5}$.

(b) Use the depth- first search algorithm to produce a spanning tree of maximum height, by choosing 'a' as the root.



(c) Use Kruskal's Greedy algorithm to find the minimum weighted spanning tree for the following weighted graph:



06. (a) Let $\chi(G)$ and $\chi'(G)$ be chromatic number and chromatic index of a graph G respectively.

Find $\chi(G)$ and $\chi'(G)$ of the Peterson graph.

Draw the graphs G_1, G_2 and G_3 such that

(i) $\chi(G_1)=3$ (ii) $\chi'(G_2)=5$ (iii) $\chi(G_3)=\chi'(G_3)=4$

(b) Schedule the final exam for APU1140, APU1141, APU1142, APU2140, APU2141, APU2142, APU2143, and APU2144, using the fewest number of different time slots, if there are students taking both APU1140 and APU2144, both APU1141 and APU2144, both APU2140 and APU2141, both APU2140 and APU2142, both APU1140 and APU1141, both APU1140 and APU1142, both APU1142 and APU2140.

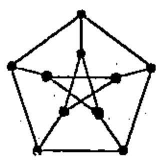
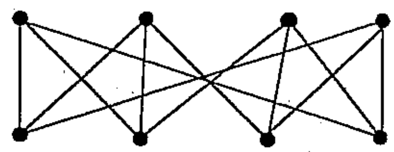
07. (a) State the Euler's Theorem.

Suppose that a connected planar simple graph has 20 vertices, each of degree 3. Into how many regions does a representation of this planar graph split the plane?

Determine the crossing number of the following graphs:

(i) $K_{3,3}$ (ii) K_5 (iii) W_5

(b) Which of the following graphs are planar? Find K_5 or $K_{3,3}$ configurations in the non-planar graphs:

(i)  (ii) 

08. Let $d(x, y)$ be the minimum length among all $x-y$ walks in a digraph D.

Let the following table be the adjacency list of a digraph $D_1 = (D(V_1), D(A_1))$.

Vertex	Adjacent Edge
t	w
u	x , t
v	t, z
w	z, v
x	y, w
y	v, x , z
z	-

(i) Draw the digraph D_1,

(ii) Find $d(u, a)$ for all $a \in V_1$,

(iii) Find $d(a, z)$ for all $a \in V_1$,

Hence deduce that D_1 is not strong.

(iv) Let $D_2 = (D(V_1), D(A_2))$, where $A_2 = A_1 \cup \{(z, u)\}$. Show that D_2 is strong.

Is D_2 a tournament? Justify your answer.