The Open University of Sri Lanka
B.Sc/B.Ed. Degree Programme
Final Examination 2011/2012
Level 05 - Applied Mathematics
APU 3146 – Operational Research
Duration: - Two Hours.

Date: -01.12.2012 **Time: - 9.30 a.m. – 11.30 a.m.**

Answer **FOUR** questions only.

01. (a) Consider the following payoff matrix for 2×2 two person zero sum game without any saddle point.

Player B

Player A	B_1	B_2
A_1	a_{11}	a_{12}
A_2	a_{21}	a_{22}

Write down the formulas for optimum mixed strategies of Player A and Player B and the value of the game to Player A.

(b) The Manager of a multinational company and the Union of workers are preparing to sit down at the bargaining table to work out the details of a new contract for the workers. Each side has developed certain proposals for the contents of the new contract. Let us call Union proposals "Proposal *1*", "Proposal *2*" and "Proposal *3*", and Manager's proposals "Contract *A*", "Contract *B*" and "Contract *C*". Both parties are aware of the financial aspects of each proposal–contract combination. The reward matrix is:

Proposal	**Contract**		
	A	***B***	***C***
1	8.5	7.0	7.5
2	12.0	9.5	9.0
3	9.0	11.0	8.0

(i) Is there any saddle point?

(ii) Find the strategies which are dominated by other strategies, and reduce the size of the reward matrix.

(iii) Formulate the Linear Programming problem to determine the optimum strategy of the Union and the optimum strategy of the Manager.

02. Your mathematics test is tomorrow, and will cover Number Theory, Linear Algebra and Vector Analysis. You must decide how to allocate your eight hours of study time among the three topics. If you were to spend the entire eight hours on any one of these topics (thus using a pure strategy) you feel confident that you will earn a 100% score on that portion of the test, but will not do so well on the other topics. You have come up with the following table, where the entries are your expected scores:

Your strategies ↓	Number Theory	Linear Algebra	Vector Analysis
Number Theory	100	50	40
Linear Algebra	50	100	50
Vector Analysis	40	50	100

Formulate a linear programming model for the above situation in order to find the expected time that you should spend on each topic.

03. On average 96 patients per 24 hour day require the service of an emergency clinic. Also on average, a patient requires 10 minutes of active attention. Assume that the facility can handle only one emergency at time. Suppose that it costs the clinic Rs.100 per patient treated to obtain an average servicing time of 10 minutes and that each minute of decrease in this average time would cost Rs.10 per patient treated. How much would have to be budgeted by the clinic to decrease the average size of the queue from $\frac{4}{3}$ patients to $\frac{1}{2}$ patient?

04. A local one person barber shop can accommodate a maximum of 5 people at a time (4 waiting and 1 getting hair cut). Customers arrive according to a Poisson distribution with mean 5 per hour. The barber cuts hair at an average rate of 4 per hour (Exponential service time)

(i) What percentage of time is the barber idle?

(ii) What fraction of the potential customers are turned away?

(iii) What is the expected number of customers waiting for a haircut?

(iv) How much time can a customer expect to spend in the barber shop?

05. There are two clerks in a university to receive dues from the students. One clerk handles with 1st and 2nd year students and the other clerk handles with 3rd and 4th year students. It has been found that the service time distributions for students are exponential with mean service time 5 minutes per students. 3rd and 4th year students are found to arrive in a Poisson fashion throughout the day with mean arrival rate 8 per hour. 1st and 2nd year students also arrive in a Poisson fashion with mean arrival rate 6 per hour.

(a) What would be the effect on the average waiting time for students if each clerk could handle any student who comes from any year?
(b) What would be the effect if this could only be accomplished by increasing the service time 6 minutes?

06. A shipping company has a single unloading landing place with ships arriving in a Poisson fashion at an average rate of three per day. The unloading time distribution for a ship with n unloading crews is found to be exponential with average unloading time $\frac{1}{2}n$ days. The company has a large labor supply without regular working hours, and avoids long waiting lines. The company has a policy of using as many unloading crews on a ship waiting in line or being unloaded.

(a) Under these conditions what will be the average number of unloading crews working at any time?

(b) What is the probability that more than 4 crews will be needed?