THE OPEN UNIVERSITY OF SRI LANKA
B.Sc./B.Ed. Degree Programme, Continuing Education Programme
APPLIED MATHEMATICS – LEVEL 05
AMU 3189/ AME 5189- STATISTICS II
FINAL EXAMINATION - 2011/12

Duration: Two Hours.

Date: 30.11.2012	Time: 9.30a.m – 11.30 a.m.

Non programmable calculators are permitted. Statistical tables are provided.

Answer FOUR questions only.

1.

A random variable X is distributed as $N(\mu,16)$. Suppose we know that μ can take only one of the two values 2 and 4. A random sample $X_1, X_2, X_3, \ldots X_{20}$ is drawn from the distribution to test the simple null hypothesis $H_0 : \mu = 2$ against the simple alternative hypothesis $H_1 : \mu = 4$.

(i) Using the Neyman- Pearson Lemma, show that the critical region of significance level 0.025 to test the above hypothesis is given by $\overline{X} \geq 3.75$.

(ii) Following table gives you a random sample of size 20 drawn from the above distribution. Test the above mentioned hypothesis and give your conclusion at 0.025 significance level.

-1.78	2.46	0.58	3.12	1.7
-0.27	5.6	5.84	8.37	9.73
-2.38	0.36	-1.36	3.13	7.32
7.29	-2.43	3.3	-0.94	0.11

(iii) Calculate the power of the test.

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2.

A box containing mangoes has an unknown proportion p of the mangoes which are sour. A random sample of n mangoes was drawn with replacement. Suppose X of them were found to be sour.

$$\text{Let} \quad Y_i = \begin{cases} 1 & ; \quad \textit{if the } i^{th} \textit{ mango is sour} \\ 0 & ; \quad \textit{Otherwise} \end{cases}$$

(i) Show that $E(X) = np$ and $var(X) = np(1-p)$.

(ii) Derive a method of moments estimator for p.

(iii) Is the moment estimator derived by you in (ii) an unbiased estimator? Prove your answer.

(iv) Obtain maximum likelihood estimator for p.

(v) Obtain maximum likelihood estimators for $E(X)$ and $Var(X)$.

(vi) Suppose 200 mangoes were tested from the above box and 20 of them were found to be sour. The box contains 1000 mangoes. Estimate the total number of sour mangoes in the box using the estimator in part (iv).

3.

(a) Briefly explain the following.

(i) Sampling distribution

(ii) Accuracy and precision of an estimator

(iii) Properties of maximum likelihood estimators

(b) Let $X_1, X_2, X_3, \ldots X_n$ be a random sample from a density function

$$f(x;\theta)\,; \quad -\infty \le x \le \infty, \; 0 \le \theta \le b$$

Let $\widehat{\theta}_1, \widehat{\theta}_2, \widehat{\theta}_3, \widehat{\theta}_4$ be functions of $X_1, X_2, X_3, \ldots X_n$. Suppose $\widehat{\theta}_3, \widehat{\theta}_4$ are unbiased estimators for parameter θ and $\widehat{\theta}_1$ is an asymptotically unbiased estimator for parameter θ. Assume that

$$Var(\widehat{\theta}_1) = \frac{n+\theta}{n^2}; \quad Var(\widehat{\theta}_2) = \frac{n+\theta}{3n}; \quad Var(\widehat{\theta}_3) = \frac{n+\theta}{n}; \quad Var(\widehat{\theta}_4) = \frac{n+\theta}{2n}$$

(i) A student says that estimator $\widehat{\theta}_4$ is better than estimator $\widehat{\theta}_3$. Do you agree with the student? Justify your answer.

(ii) Among the statistics $\widehat{\theta}_1, \widehat{\theta}_2, \widehat{\theta}_3, \widehat{\theta}_4$ which estimator do you select as the best estimator for θ. Justify your answer.

(iii) Which of the following statistics are unbiased estimators for θ? Prove your answer.

a) $\frac{2\widehat{\theta_1}+\widehat{\theta_3}+\widehat{\theta_4}}{4}$

b) $\frac{2\widehat{\theta_3}-\widehat{\theta_4}}{2}$

4.

Let $X_1, X_2, X_3, \ldots X_n$ be a random sample from a uniform distribution with density given by

$f(x;\theta) = \theta \quad ; 0 \leq x \leq {}^{1}/_{\theta} \; ; \quad \theta > 0$

(i) Find the mean and the variance of the above distribution.

(ii) Derive a moment estimator for θ. Is the moment estimator derived by you an unbiased estimator for θ? Prove your answer.

(iii) Derive maximum likelihood estimators for mean and the variance of the above distribution.

(iv) A sample drawn from the above distribution is given in the following table. Find estimates for the mean and the variance for the above distribution using the maximum likelihood estimators derived in part(iii).

0.29	1.48	0.23	1.9
0.86	1.77	0.18	1.13
1.2	0.29	0.01	0.96
1.45	1.47	0.02	1.68
1.88	1.81	1.55	0.13

5.

During the first three months of 2012, a technician was timed for the repair of an electronic instrument on 12 separate occasions. In the same period, a trainee technician was timed for the repair of a similar instrument on 14 occasions. These times, in minutes, are given in the table below.

Technician	314	296	300	324	255	242	374	250	307	215	317	334		
Trainee	279	351	282	280	258	267	312	357	322	249	228	315	311	341

Assume that these observations may be regarded as independent random samples from normal populations with equal standard deviation of 40 minutes.

(i) Construct a 95% confidence interval for the mean time difference taken to repair the instrument by technician and trainee. Interpret your answer.

(ii) Does the data provide evidence to justify the claim " The time taken to repair the instrument is the same by the technician and the trainee"

(iii) Subsequently it was learned that the times for the trainee and the technician were incorrectly recorded and that each of the values above is 30 minutes less than the actual. What, if any difference does this make to the result of the part(i).

6.

(a) Briefly explain the following.

(i) Point Estimation.

(ii) Interval Estimation.

(b) The manager of a lemonade bottling plant is interested about the performance of a production line which has only recently been installed. The manger has selected 20 one hour periods at random and has recorded the number of crates completed in each hour by this line. The table below gives the results.

77	80	86	84	86	77	77	78	86	76
79	79	83	77	82	75	78	77	75	84

(i) Construct a 95% confidence interval for the variance of the number of crates completed in each hour by the new line. Interpret your results.

(ii) Suppose the mean no of crates completed in an hour by another old line is 80. The manager claims that the "mean no of crates completed in an hour by the new and the old lines are same". Construct a 95% confidence interval for the mean number of crates completed in an hour by the new line. Comment on the manager's claim.