The Open University of Sri Lanka

B.Sc. /B.Ed. Degree Programme

Final Examination - 2011/2012

Applied Mathematics – Level 5

AMU 3186 /AME 5186 – Quantum Mechanics

Duration: - Two hours

Date:- 06.12.2012 **Time: - 1.30 p.m.- 3.30 p.m.**

Answer **Four** Questions Only.

1. (i) Show that, in the Compton effect, the relation between the electron recoil angle ϕ and the photon scattering angle θ is $\tan\phi = \dfrac{\cot\frac{\theta}{2}}{1+\frac{E_0}{m_0c^2}}$, where E_0 the initial energy of the photon, m_0 the rest mass of the electron, c the velocity of the light. The relativistic energy-momentum relation $E = \left(m^2c^4 + p^2c^2\right)^{\frac{1}{2}}$ for the electron's energy E may be used, if necessary.

(ii) Prove that the energy lost by the photon after the scattering is given by

$$E_0 - E = \frac{\left(\frac{2E_0^2}{m_0c^2}\right)\sin^2\frac{\theta}{2}}{1+\left(\frac{2E_0}{m_0c^2}\right)\sin^2\frac{\theta}{2}}.$$

2. The wave function $\psi(x)$ of a particle, free to move on a straight line ox, is given by,

$$\psi(x) = \begin{cases} A\sin\left(\frac{3\pi x}{a}\right) & |x| \le a \\ 0 & |x| > a \end{cases}$$

(i) If ψ is normalized calculate A.

(ii) Find the expectation values of the position and of momentum of the particle.

(iii) State Heisenberg Uncertainty principle.

Find the uncertainty of the momentum and of the position of the particle.

3. (i) Define the linear operator.

(ii) Operators $\hat{L}_1, \hat{L}_2, \hat{L}_3$ are defined to act as follows upon an arbitrary function $f(x)$:
$\hat{L}_1$ replaces x by $-x$, $\hat{L}_2$ squares $f(x)$ and $\hat{L}_3$ averages f over an interval $2a$ centered about x.

(a) Write down a mathematical expression for each $\hat{L}_i f(x)$; $i = 1,2,3$.

(b) Which of $\hat{L}_i$ are linear operators?

(c) Show that $\hat{L}_1$ is a Hermitian operator, and find the eigen values of that operator.

4. (a) Show that $\frac{d\langle \hat{A}\hat{B}\rangle}{dt} = \left\langle \frac{\partial \hat{A}}{\partial t}\hat{B}\right\rangle + \left\langle \hat{A}\frac{\partial \hat{B}}{\partial t}\right\rangle + \frac{i}{\hbar}\left\langle [\hat{H},\hat{A}]\hat{B}\right\rangle + \frac{i}{\hbar}\left\langle \hat{A}[\hat{H},\hat{B}]\right\rangle$, where $\hat{H}$ is the time independent Hamiltonian for the quantum system.

(b) With usual notation, show that $\frac{d}{dt}\langle rp\rangle = \langle 2\hat{T}\rangle - \langle \hat{r}.\nabla\hat{V}\rangle$.

(Provided that $\frac{\partial \hat{r}}{\partial t} = 0$ and $\frac{\partial \hat{p}}{\partial t} = 0$.)

5. A Particle of mass m and energy E moves in the positive x direction and meets a potential step which is given by

$$V(x) = \begin{cases} 0 & x < 0 \\ V_0 > 0 & 0 \le x \le a \\ 0 & a < x \end{cases}$$

where $V(x)$ is the potential energy of the particle.

Solve the Schrödinger equation for the case $V_0 < E$ and show that the transmission coefficient T given by $T = \left\{1 + \frac{V_0^2}{4E(E-V_0)}\sin^2(ka)\right\}^{-1}$, where $k^2 = \frac{2m}{\hbar}(E - V_0)$.

6. The angular momentum of a particle is defined as a vector $\underline{L}$, such that $\underline{L} = \underline{r} \times \underline{p}$ where $\underline{p}$ is the momentum and $\underline{r}$ is the position vector of the particle with respect to a fixed origin O. Write down the Cartesian components $\hat{L}_x$, $\hat{L}_y$, $\hat{L}_z$ of the angular momentum operator. Hence obtain the angular momentum operator in polar coordinates (r, θ, ϕ).

Hence prove the relation $\left[\hat{L}_x, \hat{L}_y\right] = i\hbar\hat{L}_z$.