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The Open University of Sri Lanka
B.Sc. / B.Ed. Degree Programme – Level 05
Final Examination – 2011/2012
Applied Mathematics
AMU 3184/AME 5184 – Dynamics

Duration :- Two Hours

Date :- 26.11.2012 **Time:- 01.30 p.m. –03.30p.m.**

Answer Four Questions Only.

1. (a) Obtain the components of the velocity and acceleration in intrinsic coordinates, of a particle moving along a curve in a plane.

   (b) A particle is projected with velocity v from cusp of a smooth inverted cycloid down the arc. Show that the time taken to reach the vertex is $2\sqrt{\frac{a}{g}}\tan^{-1}\left(\frac{\sqrt{4ag}}{v}\right)$.

2. (a) Show that in spherical polar coordinates, the components of the velocity and acceleration are given by $\dot{\underline{r}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} + r\sin\theta\dot{\phi}\underline{k}$ and

$$\ddot{\underline{r}} = \left(\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta\right)\hat{r} + \left(\frac{1}{r}\frac{d}{dt}\left(r^2\dot{\theta}\right) - r\sin\theta\cos\theta\dot{\phi}\right) + \frac{1}{r\sin\theta}\frac{d}{dt}\left(r^2\sin^2\theta\dot{\phi}\right)\hat{\phi}$$

   respectively.

   (b) A particle is projected horizontally with velocity u along the interior surface of a smooth hemisphere whose axis is vertical and whose vertex is downwards. The radius through the point of projection makes angle β with the downward vertical. If the particle just ascend to the point of projection show that $u = \sqrt{2ag\sec\beta}$, where a is the radius of the hemisphere.

3. (a) Obtain, in the usual notation, the equation $\frac{\partial^2 r}{\partial t^2} + 2\underline{\omega} \times \frac{\partial \underline{r}}{\partial t} = -g\underline{k}$ for the motion of a particle relative to the rotating earth.

   (b) An object is projected vertically downward with speed v_0 from a point O, near the surface of the earth, having latitude λ. Prove that after time t, the object is deflected east of the vertical by an amount $\omega v_0 \cos\lambda\, t^2 + \frac{1}{3}\omega g t^3 \cos\lambda$, where ω is the angular speed of earth about its polar axis.

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If O is at a height h above the earth, show that the particle will reach the surface of the earth, at a point east of the vertical at a distance

$$\frac{\omega\cos\lambda}{3g^2}\left(\sqrt{v_0^2+2gh}-v_0\right)^2\left(\sqrt{v_0^2+2gh}+2v_0\right).$$

4. (a) With the usual notation, show that the Lagrange's equations of motion for a holonomic system are given by $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right)-\frac{\partial L}{\partial q_j}=0,\ \ j=1,2\ldots,n.$

 (b) A uniform circular hoop of mass M and radius a swings in a vertical plane about a point O of itself, which is freely hinged to a fixed support and a bead of mass m is free to slide along the hoop. Taking C as the centre of the hoop, B as the bead and θ, d ϕ respectively as the inclinations of OC and CB to the downward vertical, obtain two equations of motion for the generalized coordinates θ and ϕ.
 If the system is gently and slightly disturbed from its position of stable equilibrium, derive the equations

$$(2M+m)\ddot{\theta}+m\ddot{\phi}+(M+m)n^2\theta=0,$$

$$\ddot{\theta}+\ddot{\phi}+n^2\phi=0 \qquad \text{where } n^2=\frac{g}{a}.$$

5. (*a*) Derive Euler's equations of motion of a rigid body rotating about a fixed point.

 (*b*) If a body moves under no forces about a point O and if H is the angular momentum about O and T the kinetic energy of the body then show that H and T are conserved.

 (c) A rigid body moves about a point O under no forces. The principal moments of inertia of a body at O being $3A$, $5A$, $6A$. Let $\underline{\omega}=(\omega_1, \omega_2, \omega_3)$ be the angular velocity of the body, at any time t, about the principal axes at O. Initially $\omega_1=n$, $\omega_2=0$ and $\omega_3=n$. Show that $\omega_2=\frac{3n}{\sqrt{5}}\tanh\left(\frac{nt}{\sqrt{5}}\right)$.

6. (a) Define the Hamiltonian H of a holonomic system and derive in the usual notation, Hamilton's equations of motion, $\frac{\partial H}{\partial p_i}=\dot{q}_i,\ \ \frac{\partial H}{\partial q_i}=-\dot{p}_i$.

 (b) The Hamiltonian of a dynamical system is given by $H=qp^2-qp+cp$ where c is a constant. Obtain Hamilton's equations of motion and hence find p and q at time t.