The Open University of Sri Lanka
B.Sc. /B.Ed. Degree Programme
Final Examination 2011/2012
Level 04 - Applied Mathematics
AMU 2185/AME 4185 – Numerical Methods I

Duration :- Two Hours

Date :- 19.11.2012 **Time:- 09.30 a.m. – 11.30 a.m.**

Answer Four Questions Only.

1. (a) Briefly explain the following :

(i) Truncation error,

(ii) Rounding error,

(iii) Relative error.

(b) Discuss the error of the value which is obtained by multiplying the values of x^{α} and y^{β} in terms of errors of x and y, where α and β are real numbers.

(c) The function u is defined as $u=\frac{5xy^2}{z^3}$. Find the relative error of u at (1, 1, 1) given that the maximum error of each variable be 0.001.

2. (a) Describe the geometrical interpretation of the Regula-Falsi formula for solving a non-linear equation $f(x)=0$.

(b) (i) The root of the equation $x\log_{10}x-1.2=0$ lies between 2 and 3. Using the method of Falsi position, find this root correct to 5 decimal places.

(ii) Estimate the number of iterations that will be required to find a solution to $x\log_{10}x-1.2=0$, in the interval $[2,3]$ correct to 5 decimal places by using the of method of bisection.

3. (a) Describe the geometrical interpretation of Newton's Formula for solving a non-linear equation $f(x)=0$.

(b) With the usual notation, prove that a condition for convergence of Newton's method is $|f(x^*)f''(x^*)| < |f'(x^*)|^2$, where x^* is a solution.

(c) Show that the equation $x-\frac{1}{(x+1)^2}=0$ has a real root in (0, 1). Derive a simple iterative scheme that can be expected to converge to this root. To what decimal place will the result of iterations be accurate, if iterations are stopped at convergences to three decimal places?

4. (a) Derive Horner's Scheme for division of a monic n^{th} degree polynomial $f(x)=x^n+a_1x^{(n-1)}+\ ...+a_{n-1}x+a_n$ by a factor $(x-\alpha)$.

(b) Let $f(h)=h^3+2h^2+10h-20$.

(i) Find all zeros of $f(h)$ correct to 3 decimal places.

(ii) What is the maximum value of $f(h)$ for $h>0$?

5. (a) Derive Newton's Backward Formula.

(b) The Tidal depth in sea at fixed lengths are given as follows :

Length (m)	5	10	15	20	25
Depth (km)	0.42	0.50	0.38	-	0.96

(i) Find the missing value,

(ii) Then use the Newton's backward method to estimate the tidal depth at length 22m.

(c) In the usual notation, prove the following:

(i) $\nabla = 1 - E^{-1}$,

(ii) $\Delta = E - 1$,

(iii) $E\nabla = \Delta$.

6. (a) (i) Write down the n^{th} order Lagrange's interpolation polynomial $p(x)$ for the data set $(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)$.

(ii) Write down the error of interpolation in the Lagrange's method.

(b) Some values of the function $y = \cos\frac{x\pi}{4}$ are tabulated as follows:

x	0	1	2
y	1	$\frac{1}{\sqrt{2}}$	0

(i) Find the Lagrange's curve $y = p(x)$ through these points.

(ii) Find the maximum absolute error of using $p(x)$ in place of the given function.