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The Open University of Sri Lanka
B.Sc./B.Ed Degree Programme – Level 04
Final Examination 2011/2012
Applied Mathematics
APU 2142/ APE4142 – Newtonian Mechanics I

Duration :- Two Hours

Date :-22.11.2012 **Time:-01.30 p.m. 03.30 p.m.**

Answer Four Questions Only.

1. (i) With the usual notation show that for a particle moving along a curve, the velocity and acceleration components in intrinsic coordinates are given by

$\underline{v} = \dot{s}\,\underline{t}$ and $\underline{a} = \ddot{s}\,\underline{t} + \frac{\dot{s}^2}{\rho}\underline{n}$ where $\underline{t}$ is the unit vector in the direction of the tangent and $\underline{n}$ is the unit vector in the direction of the inward normal.

(ii) A smooth wire in the form of an arch of the cycloid, with intrinsic equation $s = 4a\sin\psi$, is fixed in a vertical plane with its vertex downwards. The tangent at the vertex is horizontal. A small bead, of mass m, is threaded onto the wire and is subject to an air resistance of magnitude $\frac{mv^2}{8a}$ when its speed is v, this resistance being always directly opposite to the direction of motion.

Given that the bead is projected from the vertex with speed $\sqrt{8ag}$. Show that the bead comes to instantaneous rest at a cusp where $s = 4a$.

2. (i) With the usual notation show that for a particle moving in a plane, the velocity and acceleration components in plane polar coordinates are given by $\underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta$ and

$\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + \frac{1}{r}\frac{d(r^2\dot{\theta})}{dt}\underline{e}_\theta$.

(ii) Two particles A and B, each of mass m, are attached to the ends of a light inextensible string of length $2a$. The string passes through a smooth ring fixed at the point O on a smooth horizontal table. Initially the particles lie at rest on the table with $OA = OB = a$ and AOB a straight line l. Particle A is projected with speed u perpendicular to OA.

If (r, θ) are the polar coordinates of A at time t relative to O as the pole and l as the initial line and T is the tension in the string, write down equations to determine T, r and θ. Further show that:

(a) $2\frac{d^2r}{dt^2}-\frac{a^2u^2}{r^3}=0$ (b) $2r\frac{dr}{dt}=u\sqrt{2(r^2-a^2)}$ (c) $r^2=a^2+\frac{1}{2}u^2t^2$.

3. (i) With the usual notation show that the equation of the orbit of a particle moving under a central force F per unit mass is given by $\frac{F}{h^2u^2}=u+\frac{d^2u}{d\theta^2}$.

(ii) A particle P moves in a path with polar equation $r=\frac{2a}{2+\cos\theta}$ with respect to a pole O and initial line OA. At any time t during the motion $r^2\dot{\theta}=h$ (constant). Determine the central force.

4. (i) Establish the formula $\underline{F}(t)=m(t)\frac{d\underline{v}}{dt}+\underline{u}\frac{dm}{dt}$ for the motion at time t, of a particle of varying mass $m(t)$ moving with velocity $\underline{v}$ under a force $\underline{F}(t)$, matter being emitted at a rate $\frac{dm}{dt}$ with velocity $\underline{u}$ relative to the particle.

(ii) A rocket of initial total mass M propels itself by ejecting mass at a constant rate μ per unit time with speed u relative to the rocket. If the rocket is initially at rest directed vertically upwards, show that it will not initially leave the ground unless $\mu u > Mg$. Assuming this condition to hold, show that the velocity of the rocket after time t is given by $u=\ln\left(1-\frac{\mu t}{M}\right)-gt.$

Show also that when the mass of the rocket has been reduced to half the initial value, its height above the ground will be

$$\frac{uM}{2\mu}\left(1-\ln 2-\frac{Mg}{4\mu u}\right)$$

5. (i) Let $\underline{H}$ be the angular momentum of the system of particles about a fixed point O and let $\underline{M}$ be the total moment about O of the external forces acting on the system. Show that $\frac{d\underline{H}}{dt} = \underline{M}$.

(ii) A uniform solid cylinder of radius a and mass m can rotate freely about a smooth, fixed, horizontal axis which coincides with the axis of the cylinder. A light string passes over the cylinder in a vertical plane perpendicular to the axis of rotation. Particles of masses $m/2$ and m are attached one to each end of the string, and the system is released from rest. Assuming that the string does not slip on the cylinder and that neither particle has reached the cylinder, calculate the tensions in the two parts of the string and the angular acceleration of the cylinder.

6. (i) Show that the impulsive moment of the resultant force about an axis is equal to the gain of angular momentum.

(ii) A uniform rod AB of mass m and length $2a$ rests on a smooth, horizontal table and is free to rotate about a vertical axis through the end A. The rod receives a blow from a hammer at its free end in a direction perpendicular to the rod and begins to rotate with angular velocity ω. Calculate the impulse of the blow.