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The Open University of Sri Lanka
B.Sc. /B.Ed. Degree Programme
Final Examination 2011/2012
Level 04 - Applied Mathematics
AMU 2184/AME 4184 – Newtonian Mechanics

Duration :- Two Hours

Date :- 22.11.2012 **Time:- 1.30 p.m. - 3.30 p.m.**

Answer **Four** Questions Only.

01. A particle P of mass m is projected from a point O, vertically upwards under gravity in a medium which exerts a resisting force of magnitude mgv^2, where v is the speed of the particle. If U is the speed of projection, show that the greatest height of P above O is $\frac{1}{2g}\ln\left(1+U^2\right)$.

If V is the speed of P on returning to O, show that $\frac{1}{V^2}-\frac{1}{U^2}=1$.

02. (a) Establish the formula $\underline{F}(t)=m(t)\frac{d\underline{v}}{dt}+\frac{dm}{dt}\underline{u}$ for the motion of a particle of varying mass $m(t)$ with velocity $\underline{v}$ under a force $\underline{F}(t)$, matter being emitted at a constant rate with velocity $\underline{u}$ relative to the particle.

(b) At time t, a rocket with fuel of mass $m_0(1-kt)$ is moving with speed v vertically upwards. The fuel is ejected with a constant exhaust speed u and at a constant rate k. Write down the equation of motion of the rocket, assuming that air resistance can be neglected and that the height of the rocket above the earth's surface is always small enough so that the gravitational effect of the earth could be considered as producing a uniform acceleration.
Find the velocity v and the distance travelled x when the mass of the rocket is half of the original value.

03. (a) With the usual notation, show that the velocity and acceleration components in plane polar coordinates are given by $\underline{v}=\dot{r}\underline{e}_r+r\dot{\theta}\underline{e}_\theta$ and $\underline{a}=\left(\ddot{r}-r\dot{\theta}^2\right)\underline{e}_r+\frac{1}{r}\frac{d}{dt}\left(r^2\dot{\theta}\right)\underline{e}_\theta$.

(b) A particle A, of mass m is held at rest on a smooth horizontal table. One end of a light inextensible string is attached to A. The string passes through a small smooth hole O in the table, and carries at the other end a particle B also of mass m hanging freely. Initially $OA=a$ and the particle A is moving horizontally with speed $\sqrt{2gh}$, where $h>a/2$, in a direction perpendicular to the string. If r is the distance OA after time t, show that $\dot{r}^2=gh\left(1-\frac{a^2}{r^2}\right)+g(a-r)$.

04. A catapult consists of a sling attached to two elastic strings, each of natural length $2a$ and modulus of elasticity $2mg$. The free ends of the strings are attached to a frame at a distance $4a$ apart. The catapult is used to project to a particle of mass m vertically upward. The sling is drawn back until the strings are each of length $8a$ and then released. Find the projected speed of the particle.

05. (a) With the usual notation, prove that the equation of the central orbit of a particle moving under central force F per unit mass is $\frac{d^2u}{d\theta^2}+u=\frac{F}{h^2u^2}$ where $u=\frac{1}{r}$.

(b) The path of a particle moving under a central force is given by $r=a(\cos\theta+1)$. Find the law of force.

06. The earth's attraction varies inversely as the square of the distance from its centre, and mg is its magnitude at the surface. Show that the time of falling from a height h above the surface of the earth to the surface is $\sqrt{\frac{a+h}{2g}}\left[\sqrt{\frac{h}{a}}+\frac{a+h}{a}\sin^{-1}\sqrt{\frac{h}{a+h}}\right]$,

where a is the radius of the earth and the resistance of the air is neglected. If h is small compared with a, show that this result is approximately $\sqrt{\frac{2h}{g}}\left[1+\frac{5h}{6a}\right]$.