The Open University of Sri Lanka
B.Sc./ B.Ed. Degree/ Continuing Education Programme
Level-04 Final Examination-2011/2012
PUU 2142/PUE4142-Linear Algebra
Pure Mathematics

Duration: Two Hours.

Date: 11-01-2012.	**Time: 09.30 a.m. – 11.30 a.m.**

Answer FOUR questions only.

1. (i) Show that $\left(A^{-1}BA\right)^2 = A^{-1}B^2A$. Generalize to $\left(A^{-1}BA\right)^n$.

(ii) Let $A=\begin{pmatrix} a & b & b & b \\ b & a & b & b \\ b & b & a & b \\ b & b & b & a \end{pmatrix}$. Show that $|A|=(a+3b)(a-b)^3$.

(iii) Find the inverse of the matrix B where

$$B=\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

(iv) Find non-singular matrices P and Q such that PAQ is of the normal form,

$$\text{where} \quad A=\begin{pmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{pmatrix}.$$

Hence find the rank of A.

2. (i) Define the consistent and inconsistent systems.

   (ii) If there are n homogeneous linear equations in n unknowns, then prove that the system has only the trivial solution if the co-efficient matrix is non-singular.

   (iii) Find the rank of the matrix A where

$$A = \begin{pmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{pmatrix}.$$

   (iv) Solve the following systems of simultaneous linear equations;

   (a)
$$\begin{aligned} x + y + z &= 1 \\ x - y + z &= 3 \\ x \quad + z &= k \end{aligned}$$

   (b)
$$\begin{aligned} t + x + y + z &= 1 \\ t - x - y + z &= 0 \\ 2t + x + y - z &= 2. \end{aligned}$$

3. (i) State the Cramer's Rule.

   (ii) Use the Cramer's Rule (if applicable) to find the solutions of the following system of linear equations.

$$\begin{aligned} x + 2y - 5z + 2w &= -2 \\ 3x - y + 2z + 4w &= 19 \\ 2x + 3y - 4z + 3w &= 6 \\ 5x + \; y \; + z \; - w &= 3 \end{aligned}$$

   (iii) Use the Cayley Hamilton Theorem to compute A^{-1}, $A^{-2,}$ A^{3} and A^{4} if

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{pmatrix}.$$

4. (i) Let A be a matrix having eigen values λ_i where i=1, 2, 3,⋯. Prove that the matrix kA has eigen values $k\lambda_i$ where k is a constant.

   (ii) Diagonalize each of the following matrices and find the relevant modal matrices.

   (a) $\begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$ (b) $\begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

   (iii) Find A^4 for each of the above matrices.

5. (i) Show that any square matrix A can be written as the sum of a symmetric matrix B and skew-symmetric matrix C.

   (ii) Find the orthogonal transformation which transforms the quadratic form

   $3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_2x_3 + 2x_1x_3 - 2_1x_2$ to its canonical form.

6. Solve each of the following systems by LU-decomposition.

   (i) $$\begin{aligned} 2x - 3y + z &= 1 \\ x + y - z &= 0 \\ x - 2y + z &= -1 \end{aligned}$$

   (ii) $$\begin{aligned} 2x + y + 5z + t &= 5 \\ x + y - 3z - 4t &= -1 \\ 3x + 6y - 2z + t &= 8 \\ 2x + 2y + 2z - 3t &= 2 \end{aligned}$$