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The Open University of Sri Lanka
B.Sc./B.Ed. Degree Programme/ Continuing Education Programme
Final Examination - 2011/2012
Pure Mathematics – Level 04
PMU2191/PME4191- Vector Analysis

Duration: Two hours

Date: 02.01.2012 **Time: 9.30 – 11.30 am**

Answer Four Questions Only.

1. (a) If $u = \ln(2x+2y) + \tan(2x-2y)$, prove that

   (i) $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} = \dfrac{2}{x+y}$,

   (ii) $\dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial^2 u}{\partial y^2}$.

   (b) Using chain rule find $\dfrac{du}{dt}$ at $t=0$ for the function $u = x^2 - y^2$, where $x = e^t \cos t$ and $y = e^t \sin t$.

   (c) Considering the equation $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$ show that $xz\dfrac{\partial^2 z}{\partial x^2} + x\left(\dfrac{\partial z}{\partial x}\right)^2 - z\left(\dfrac{\partial z}{\partial x}\right) = 0.$

   (d) Find the maximum and minimum values of the function $f(x,y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2$ and determine their nature.

2. (a) Find the equation of the tangent plane to the surface $z = 3 + \dfrac{x^2}{16} + \dfrac{y^2}{9}$ at the point $(-4,3,5)$.

   (b) Using Taylor expansion of a suitable multi variable function, approximately evaluate $2.36^2\sqrt{5.68^2 + 4.10^3}$.

   (c) (i) Find the maximum rate of change of the scalar field $f(x,y) = xy + yz + zx$ at the point $(2,-1,1)$ and the direction in which this maximum occurs.

   (ii) Obtain the rate of change of the scalar field f at the point $(1,1,-2)$ in the direction of the vector $\underline{u} = \underline{i} + 2\underline{j} + 2\underline{k}$.

3. (a) Show that $\underline{\nabla}\phi$ is a vector perpendicular to the surface $\phi(x,y,z)=C,$ where C is a constant.

(b) Find the unit vector normal to the surface $x^2-y^2+z=2$ at the point $(1,-1,2)$.

(c) If $\underline{A}=x^2z\underline{i}-2y^3z^2\underline{j}+xy^2z\underline{k},$ then find $div\underline{A}$ and $Curl\underline{A}$ at the point $(1,-1,1)$.

(d) Let $\underline{F}(x,y,z)$ be a vector field and $\phi(x,y,z)$ be a scalar field.
Prove that $div(\phi\underline{F})=\phi\, div\underline{F}+grad\phi\cdot\underline{F}.$
Hence Show that $div\left(\frac{\underline{r}}{r^3}\right)=0,$ where $r=|\underline{r}|$ and $\underline{r}=x\underline{i}+y\underline{j}+z\underline{k}.$

4. (a) What is an irrotational vector?
Determine the constant a,b and c so that the vector
$\underline{F}=(x+2y+az)\underline{i}+(bx-3y-z)\underline{j}+(4x+cy+2z)\underline{k}$ is irrotational.

(b) If $\underline{F}=(2x+y)\underline{i}+(3y-x)\underline{j},$ evaluate $\int_C \underline{F}\cdot d\underline{r}$ where C is the curve in the $xy-$plane consisting of the straight line from $(0,0)$ to $(2,0)$ and then to $(3,2)$.

(c) Find the volume integral of the function $f(x,y,z)=z+3x-2$ over the region inside the circular cylinder $x^2+y^2=1$ and lying between the planes $z=0$ and $z=1$.

5. (a) State ***Stokes' Theorem***.

(b) If S is any open surface bounded by a simple closed curve C and $\underline{B}$ is any vector then prove that $\oint_C d\underline{r}\wedge\underline{B}=\iint_S(\underline{n}\wedge\underline{\nabla})\wedge\underline{B}\,ds.$

(c) If $\underline{A}=(x-z)\underline{i}+(x^3+yz)\underline{j}-3xy^2z\underline{k}$ and S is the surface of the cone $z=2-\sqrt{x^2+y^2}$ above the $xy-$plane then use ***Stokes' Theorem*** to evaluate $\iint_S(\underline{\nabla}\wedge\underline{A})\cdot\underline{n}\,dA.$

(d) By applying ***Stokes' Theorem*** to the vector field $\underline{F}=P(x,y)\underline{i}+Q(x,y)\underline{j}$ and a flat surface S in the $xy-$plane, bounded by the curve C, show that

$$\iint_S\left(\frac{\partial Q}{\partial x}-\frac{\partial P}{\partial y}\right)dx\,dy=\oint_C\left(P\frac{dx}{dt}+Q\frac{dy}{dt}\right)dt.$$